$$A=\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}.$$

(d) Write the matrix in row reduced Echelon form

$$A=\begin{bmatrix}1&-1&0\\1&0&1\\-1&1&1\end{bmatrix}.$$

(e) Find inverse of matrix $\begin{bmatrix}4&3\\1&-1\end{bmatrix}$.

(f) Determine the values of 'a' such that Rank of the matrix $\begin{bmatrix}a&4\\1&2\end{bmatrix}$ is 1.

(g) Check whether $f(x)=x^2+1$ is invertible.

(h) Prove that a set of vectors contains atleast one zero vector is L.D.

(i) Prove that four vectors $\alpha_1=(1,0,0)$, $\alpha_2=(0,1,0)$, $\alpha_3=(0,0,1)$ and $\alpha_4=(1,1,1)$ in v_3 form a L.D. set.

(j) If 'a' is a divisor of two integers g and h, then $a\,|\,(mg+nh)$ for any integer m and n.

2023

Full Marks - 80

Time - 3 hrs.

The figures in the right hand margin indicate marks.

Answer ALL questions.

Part – I

1. Answer all : [1 x 12 = 12

(a) Determine principal argument of Z = 1 – i.

(b) Represent Z = 3i in Polar form.

(c) Magnitude of Complex Number $Z=\frac{1}{1+i}$ is ________.

(d) If R is the equivalence relation defined in the set A, then equivalence classes are non-empty sets. (T/F)

(e) The smallest equivalence relation defined on the set A = {1, 2, 3} is ________.

(f) If $f:R\to R$ by $f(x)=x^2+1$ and $g:R\to R$ by $g(x)=\frac{1}{x}$. Then $(fog)(2)=$ ________.

(g) A system of linear equations is said to be consistent. If there exist _______ solution.

(h) For which value of K, the system of Equations $kx+2y=0$ and $2x+ky=0$ have infinite solutions.

(i) State division Algorithm.

(j) What is Rank of the matrix $A=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$.

(k) A set of vectors contains atleast one zero vectors is L. I. (T / F)

(l) What is singular matrix.

Part – II

2. Answer any EIGHT questions : [2 x 8 = 16

(a) Put complex number $\left(\frac{2+i}{3-i}\right)^2$ in Polar form.

(b) Simplify $\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right]^{\frac{1}{4}}$.

(c) Define Equivalence Class.

(d) If $x\equiv a(\bmod n)$ and $y\equiv b(\bmod n)$, prove that $xy\equiv ab(\bmod n)$.

(e) If dimensions of subspaces 'v_1' and 'v_2' of a vector

[Cont...

space 'v' are respectively 5 and 7 and $\dim(v_1+v_2)=1$. Then find $\dim(v_1\cap v_2)$.

(f) Define Eigen values of a matrix.

(g) Determine whether the map $T:v_2\to v_2$ defined by $T(x,y)=(x^2,y)$ is linear.

(h) Show that the vectors (1, 2) and (3, 4) are L. I.

(i) State Fundamental theorem of Arithmatic.

(j) Write the vector equation as a system of equation

$$x_1\begin{bmatrix}2\\3\\-1\end{bmatrix}+x_2\begin{bmatrix}0\\1\\2\end{bmatrix}+x_3\begin{bmatrix}-2\\-3\\0\end{bmatrix}=\begin{bmatrix}11\\9\\4\end{bmatrix}.$$

Part – III

3. Answer any EIGHT questions : [3 x 8 = 24

(a) Solve the equation $z^3-1=0$ using De moivres theorem.

(b) Find smallest positive integer 'n' for which $\left(\frac{1+i}{1-i}\right)^n=1$.

(c) Find characterstics equation of the matrix

[Cont...

Part – IV

Answer all the questions : [4 x 7 = 28

4. If $z_1 + \frac{1}{z_1} = 2\cos\theta$, $z_2 + \frac{1}{z_2} = 2\cos\theta$. Show that $z_1^m\, z_2^n + \frac{1}{z_1^m\, z_2^n} = 2\cos\left(m\theta + n\varphi\right)$.

OR

Find square root of $-7 + 24i$.

5. Use Mathematical Induction to prove $10^{2n-1} + 1$ is divisible by 11.

OR

Prove that R is uncountable.

6. For what value of λ and μ the given system having (a) No solution (b) Unique solution $x_1 + x_2 + x_3 = 3$, $4x_1 + 3x_2 - 2x_3 = 5$, $2x_1 + 3x_2 + \lambda x_3 = \mu$.

OR

Investigate for consistency of following equations. If possible find the solution $4x - 2y + 6z = 8$, $x + y - 3z = -1$, $15x - 3y + 9z = 21$.

7. Find Rank and Nullity of given matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}.$$

OR

Find basis and demension for the rull space of matrix $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.