QUESTION BANK FOR UNDER GRADUATE STUDENTS 2023-24

DEAPAERTMENT OF MATHEMATICS

SHAILABALA WOMEN’S (AUTO) COLLEGE, CUTTACK-753001

PREFACE

Evaluation is an integral part of the learning process. The capacity to answer a question effectively is a measure of the progress made in learning a subject.

Questions can be set on a given topic in a variety of ways depending on the angle the topic is viewed at. The ease with which questions are answered indicates the degree of assimilation of the topic.

Objective type questions are important as they help the learner to perceive the crucial ideas. The sooner these questions are answered the better is the comprehensive of the learner.

Questions given in this Bank are by no means exhaustive though efforts have been made to include typical ones. These are only representatives and the serious student is advised to see the questions set by the great masters. Care has been taken to see that questions conform to the pattern set in examinations. Corrections, if any, and suggestions for improvement shall be gratefully received.

DEPT. OF MATHS.
Sailabala Women's College
Cuttack

CONNTENTS

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Appendix- I

Sailabala Women's College,

B.A./ B.Sc. Pass syllabus in Mathematics.

There shall be four papers each carrying hundred marks (100) and of three (3) hour duration.

Paper -I	Calculus	32 marks
	Ordinary Differential Equation	32 marks
	Analytical Solid Geometry	16 marks
Papers- II	Linear Algebra	50 marks
	Modern Algebra & Theory of Equation	50 marks
Paper- III	Analysis -I	50 marks
	Advance Calculus	50 marks
Paper-IV	Numerical Analysis	50 marks
	Operation Research	50 marks

Books Prescribed :-

1. Calculus by Shanti Naraysn (S. Chand & Co) (Part – II & III)
   Chapters : Part -II; 8 (24, 25, 26) :
2. Part - III - 1(1, 2), 3, 4 (art 10-12 omitting Simpsons rule), 5 (art.13), 6 (art.15)
3. Elements of ordinary Different Equations by J. Sinha Ray & S. Padhy. (Kalyani) Chap – 2 (2. 1 – 2.7), 3, 4 (4.1 – 4.7), 5, 9, (9.1, 9.2, 9.4, 9.5, 9.10, 9.11, 9.13)
4. Analytic Solid Geometry, by Shanti Narayana (S. Chand & Co.)
   Chap – 6(6.3, 6.3.1, 6.3.2, 6.3.3, 6.4, 6.4.1, 6.5, 6.6, 6.6.1, 6.7, 6.7.1) 7(7.1, 7.1.1, 7.1.2, 7.2, 7.4, 7.4.1, 7.4.6, 7.6, 7.6.1, 7.7, 7.7.1, 7.8, 7.8.1, 7.8.2), 8 (8.1, 8.2, 8.3, 8.3.1, 8.3.2, 8.3.3)

5. An Introduction to Linear Algebra by V. Krishnamurthy and two others. (Affiliated East –West Press),
   Chap - 3(3.1 - 3.6), 4(4.1 – 4.5), 4(4.6 , 4.7), 5(5.1 - 5.9), 6 (6.1, 6.2, 6.5, 6.6), 7(7.4 only).

6. Topics in Algebra by I. N. Herstein (Vikas)
   Chap – 1(1.3), 2(2.1 – 2.7), 3(3.1 – 3.5)

7. Algebra and Theory of Equations by Chandrika Prasad.
   Chap.- 11(11.1 – 11.4), 12(12.1-12.3, 12.6)

8. Fundamental of Mathematical Analysis, by G. Das & S Pattanaik (TMH)
   Chap – 2(2.2 – 2.4, 2.5 - 2.7), 3(3.2, 3.3, 3.4), 4 (4.1 - 4.7, 4.10, 4.11), 5 (5.1 - 5.5), 6 (6.1 - 6.7, 6.9), 7 (7.1 - 7.6)

9. Mathematical Analysis : S. C. Mallick, S. Arora & others (New Age) Chap- 15(15.1 – 15.10), 17, 18.

10. Topics in Calculus : R. K. Panda & P. K. Satapathy.

11. A course on Numerical Analysis : B. P. Acharya & R. N. Das (Kalyani), Chap.- 1, 2(2.1-2.4, 2.6, 2.8, 2.9), 3(3.1-3.4, 3.6-3.8, 3.10), 4(4.1, 4.2), 5(5.1-5.3), 6(6.1-6.3, 6.10, 6.11), 7(7.1-7.4 & 7.7).

12. Operation Research by Kantiswarup, P. K. Gupta, Man Mohan (Sultan Chand) Chap.- 0 (0.13, 0.14, 0.15), 2, 3, 4 (4.1 - 4.4) 5, 10 & 11.

-The End-

Appendix - II

B.A/ B. Sc. (Hons.) Syllabus in Mathematics.

There shall be seven (7) theory papers; each carrying hundred marks and shall be of three hours duration. There shall be one practical examination of six hours duration carrying hundred marks. Electronic calculators may be used by students inside the examination hall of the following subjects: i) Numerical Analysis, ii) Operation Research, iii) Probability Theory, iv) Discrete Mathematics, v) Financial Mathematics and vi) Computer Programming.

First Year

Paper -I	Calculus	32 marks
	Ordinary Differential Equation	32 marks
	Analytical Solid Geometry	16 marks
Papers- II	Linear Algebra	50 marks
	Modern Algebra & Theory of Equation	50 marks

Second Year

Paper- III	Analysis -I	50 marks
	Advance Calculus	50 marks
Paper-IV	Numerical Analysis	50 marks
	Operation Research	50 marks

Third Year

Paper - V	Probability	50 Marks
	Partial Differential Equation	50 Marks
Paper - VI	Analysis- II	50 Marks
	Complex Variable	50 Marks
Paper - VII	Any two elective subjects	

a) Financial Mathematics

b) Discrete Mathematics

c) Number Theory

d) Mechanics

e) Differential Geometry

f) Mathematical Modelling.

Paper - VIII PRACTICAL

Programming in 'c' with practical 100 Marks $\begin{cases} Record - & 20 \\ Viva - & 30 \\ Practical - & 50 \end{cases}$

Books Prescribed :-

1. Mechanics by J. L. Synge and Griffith (Mc Graw Hill)
2. Probability : Elementary Probability Theory with Stochastic Process, by K. L. Chung (Narosa)
3. Differential Geometry : Differential Geometry of three dimension by C. E. Weatherburn (ELBS)
4. Partial Differential Equations : A course of Ordinary and Partial Differential Equations by J. Sinha Roy, S. Padhy (Kalyani)
5. Computer Programming : Fortran - IV Programming by M. G. Chopra, Ramkumar (Vikas
6. O.R. : Linear Programming by R. K. Gupta (Krishna Prakasani Mandir)
7. Number Theory : An Introduction to the theory of Numbers by Niven, S. Zuckerman (Wiley Eastern)

Books for the other subjects are same as those prescribed for PASS course.

-The End-

11. If G_1, G_2 ......, G_n are open subsets of X, then show that $\bigcap_{k-1}^{n} G_k$ is also open.

12. If $F_1, F_2,$ ...., F_n are closed sets in X, then $\bigcup_{k=1}^{n} F_k$ is also closed in X.

13. Prove that every closed disk in X is a closed set in X.

14. Prove that if x is a limit point of S then there exists a sequence (x_n) of distinct terms in S such that $\lim_{n\to\infty} x_n = x$

15. Prove that a set $S \subset X$ is closed iff $S \supset D(S)$, where $D(S)$ is the derived set of S.

16. $S \cup D(S)$ is always a closed set.

17. Prove that every bounded infinite subset of R $(or\ C)$ has at least one limit point.

18. Prove that a subset S of X is compact iff S is sequentially compact/ Prove that ..... iff S is Cauchy complete and given $\in > 0$, there exist $x_1, x_2, \ldots, x_n \epsilon S$ such that $S \subset \bigcup_{k=1}^{n} N(x_k, \epsilon)$.

19. If f and g are real functions on the same domain and continuous at a, then $\frac{f}{g}$, g(a) ‡ 0, is also continuous at a. Prove this.

20. If f is continuous at a and g is continuous at $f(a)$ then $g\ o\ f$ is continuous at a. Prove this.

21. Let X be closed and bounded subset of R and $f: X \to R$ be continuous. Then f attains its maximum and minimum. Prove this.

22. Let $f: [a, b] \to R$ be continuous and let $m \le a \le M$ where $m = \inf_{x\in[a,b]} f(x)$ and $M = \sup_{x\in[a,b]} f(x)$ then there exists a $c \epsilon [a, b]$ such that $f(c) = a$. Prove this.

23. Prove that if $f: [a, b] \to R$ is strictly increasing and continuous on $[a, b]$ then $f^{-1}: [f(a), f(b)] \to R$ is strictly increasing and continuous.

24. Test the differentiability of the following function at the indicated points. (any one)

i) $f(x) = \begin{cases} \text{Sin } x, & x \leq \frac{\pi}{2} \\ 1 + \left(x - \frac{\pi}{2}\right)^2, & x > \frac{\pi}{2} \end{cases}$, at $x = \frac{\pi}{2}$.

ii) $f(x) = \begin{cases} x^2 \text{ Sin} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, at $x = 0$ etc.

25. If f and g are two differentiable functions then show that $fg, f+g, fog$ and $\frac{f}{g}$ are differentiable functions.

26. State and prove Rolle's Theorem.

27. State and prove Lagrange's Mean Value Theorem.

28. State and prove the generalised Mean Value Theorem.

29. Let $f^{n-1} \epsilon MV [a, b]$, $n \epsilon N$. Then there exists a $c \in (a, b)$ such that $(c) = f(a) + (b-a) f'(a) + \frac{(b-a)^2}{2!} f''(a) + ... + \frac{(b-a)^{n-1}}{(n-1)!} . f^{(n-1)}(a) + R_n$, where $R_n = \frac{(b-a)^n}{n!} f^{(n)}(c)$ / State and prove Taylor's theorem for expansion of a function.

30. Let f be twice continuously differentiable on an interval I. Let $f'(c) = 0$, for some $c \in I$. Then f has a local maximum at c if $f''(c) < 0$ and a local minimum at c if $f''(c) > 0$.

31. Determine the approximating polynomial of the function

$f: (0, 1) \rightarrow R$ defined by $f(x) = \frac{1}{\sqrt{1-x}}$ and also find the error function e_n.

SHORT QUESTIONS

1. If F is an ordered field prove that if for all $c > 0, a < b + c$, then $a \leq b$.
2. If S ≠ 0 and $S \subset R$ and a ∈ R then a Sup S = Sup (as) if a > 0 or a Sup S = inf (as) if $a < 0$.
3. Let A and B be subsets of R. Then Sup $(A + B) \leq Sup\ A + Sup\ B.$
4. If $A, B \subset R$ then inf $(A + B) \geq inf\ A + inf\ B.$
5. Find maximum, minimum, supremum, infimum of the following set whenever they exist.

   i. $S = \{\frac{n-1}{n}, n \in N\}$ or

   ii. $S = \{\frac{x^n}{|x|} < 1, x \in R, n \in N\}$ or

   iii. $\{x \in Q : x^2 > 2\}$ or

   iv. $\{\frac{1}{m} + \frac{1}{n} : m, n \in N\}$ or

   v. $\{\frac{n+1}{n}, n \in N\}$ etc.

6. Let S and T be non empty subsets of R such that for a $s \in S$ and $t \in T$, $s \leq t$. Then prove that Sup $S \leq inf\ T$.
7. If $x, y \in R$ and $x > 0$ then there exists $n \in N\ s.t. nx > y$.
8. Prove that A x A is countable if A is countable.
9. Prove that Z or Q is countable.
10. Prove that limit of a convergent sequence is unique.
11. Prove that a convergent sequence is bounded.
12. Prove that $\lim_{n\to\infty} n^{1/n} = 1$
13. Prove that $\lim_{n\to\infty} \frac{n}{a^n} = 0$ if a > 1.

14. Determine the least value of K such that for n > k, $\frac{n}{n^2+1} < 0.0001 \; or \; \frac{1}{n} + \frac{(-1)^n}{n^2} < 0.00001$.

15. Prove that $\lim_{n\to\infty} x_n = 0$ iff $\lim_{n\to\infty} |x_n| = 0$

16. Prove that a monotonic increasing sequence (x_n) bounded above is convergent.

17. Prove that a sequence (x_n) is convergent to 1 iff every subsequence of (x_n) is also convergent to 1.

18. Prove that every Cauchy sequence is bounded.

19. Find the cluster points of the sequence

i) $x_n = (1 - \frac{1}{n})$ Sin n$\frac{\pi}{2}$. ii) $(-1)^n$ + Sin n$\frac{\pi}{2}$. iii) Cos n$\frac{\pi}{3}$ etc.

20. Give statement of Ratio Test/ Root Test/ Condensation Test/ comparison Test.

21. Prove that $\lim_{n\to\infty} a_n = 0$ is a necessary but not sufficient condition for $\sum a_n$ to be convergent.

22. Give definition with examples (any one)

Open Set/ Closed Set/ Interior point/ Boundary Point/ Limit Point/ Closure of a Set.

23. Give examples of the following (with reason) (Any one)

i. An open set which is not an interval.

ii. A set which is neither open nor closed.

iii. A countable set whose closure is R.

iv. A family of closed sets whose union is not closed.

v. Two non closed sets whose union is closed etc.

24. Determine if the following set is open, closed, clopen, neither open nor closed.

i) {0} in R, ii) Q in R, iii) [0, 1] in R,

iv) Q in C, v) (-∞, a] in R, vi) R – {0} in R, vii) {1, 1/2, 1/3, ...} in R,

viii) [0, 1] U (1, 2) U {3} in R.

25. Determine the boundary of (-1, 1)/ {1, 2, 3}/ R – N/ (-1, 0) U (0, 1).

26. Examine whether mean value conditions MV [a, b] is satisfied for the following function. (Any one)

i. $f(x) = \frac{x-1}{x}$, a = 1, b = 3

ii. $f(x) = |x|$, a = 1, b = 2

iii. $f(x) = \log x$, $a = \frac{1}{2}$, b = 2

iv. $f(x) = [x]$, $a = -\left(\frac{1}{2}\right)$, b = 2

27. Prove that Z is not dense in R.

28. Give example of an unbounded oscillatory function on N.

29. Show that $\lim_{n\to\infty} (x_n + y_n) = \lim_{n\to\infty} x_n + \lim y_n$ where (x_n) and (y_n) are sequences of real numbers.

30. Give statement of Sandwich theorem.

31. Obtain limit of the following sequence (if exists) or write divergent. Also mention kind of divergence if divergent. (Any one)

i. $[(-1)^n + 1]n$,

ii. $\sqrt{n^2 + n} - n$,

iii. (1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, ...)

iv. $\frac{\sin nq}{n}, q \in Q$,

v. $\frac{n^2 + 2n + 1}{7n + 39}$

vi. $5 + 3(-1)^n$

32. Give example of sequences (x_n) and (y_n) such that

i) $(x_n)$and (y_n) are both divergent but $x_n y_n$ is convergent, or

ii) (x_n) is convergent and (y_n) is divergent but $x_n + y_n$ is convergent. or

iii) $\text{Lim } x_n = \infty$, $\lim y_n = 0$ but $\lim x_n y_n = 0$. Or

iv) $\text{Lim } x_n = 0$, $\lim y_n = 0$ but $\lim \frac{x_n}{y_n}$ oscillates.

State true or false. Give reason. (33 to 45)

33. Every real number is the limit of a sequence of irrationals.

34. Every real number is the limit of a sequence of rationals.

35. Every real number is the limit of a sequence of rationals and also a limit of sequence of irrationals.

36. If $\sum a_n$ is convergent and $\sum b_n$ is divergent then $\sum(a_n + b_n)$ is always divergent.

37. Whenever ratio test proves convergence the root test does so.

38. A monotonically increasing sequence is always divergent to $+\infty$.

39. {1, 2} is not a closed set in R.

40. If a set A is open in X then A can always be expressed as union of open sets in X.

41. If a set is not open then it is closed.

42. A is a finite set if it has no limit point.

43. (a, b) is open in R but not open in C.

44. If $\lim_{x \to a} f(x)$ exists then $\lim_{n \to \infty} f(x_n)$ exists where (x_n) is the sequence $(a, a, a, \dots)$

45. If f is continuous on a closed and bounded set A then f takes every value between $\inf_{x \in A} f(x)$ and $\sup_{x \in A} f(x)$

46. $|x - 1|$ is not differentiable at $x = 1$.

47. Give example where f and g are not differentiable but $f + g$ is differentiable.

48. Give example of a function which is continuous but not differentiable.

49. When does a function f satisfy the mean value condition on $[a, b]$?

50. State the conditions when a function f has local maximum at a point c.

ANALYSIS-II

+3, 3rd Year Hons.

1. Prove that for every $x_n \in R^+$ and every n ∈ N , there exists a unique $y \in R^+$ such that $y^n = x$.
2. If $x_n \in R, x \geq -1$ and $n \in N$, then prove that $(1+x)^n \geq 1+nx$. (Bernoulli's inequality).
3. If $a > 1$ and $0 < r < s$, where $r, s \in Q$, then prove that $\frac{a^r-1}{r} < \frac{a^s-1}{s}$.
4. If $a_1, a_2, \ldots \ldots, a_n \in R$ and $b_1, b_2, \ldots \ldots, b_n \in R$, then prove that $(\sum_{K=k}^{n} a_k\, b_k)^2 \leq (\sum_{k=1}^{n} a_k{}^2)(\sum_{k=1}^{n} b_k{}^2)$ (Cauchy – Schwarz – Bunyakovskii inequality)
5. Prove that every Cauchy sequence in R is bounded.
6. Prove that every convergent sequence is a Cauchy sequence.
7. Prove that every Cauchy sequence of real numbers is convergent. (Cauchy's Completeness Principle).
8. Prove that the sequence x_n in R defined by the recurrence relation $x_{n+2} = \frac{1}{2\,(x_{n-1}+x_n}$ Converges provided that $x_1 = x_2$
9. Prove that sequence (x_n) in R defined by $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$ $\forall\, n \in N$ is not Cauchy, not bounded , not convergent.
10. Give an example of Cauchy sequence in a set $S \subset R$ such that the sequence does not converge to a point of S. Give reasons for your answer.
11. If x_n be a Cauchy sequence in R such the $|x_{n+1} - x_n| < \frac{1}{2^n}$, then prove that the sequence $(x_{n)}$ is a Cauchy sequence and is convergent.
12. Prove that the sequence (x_n) in R, where $0 < a \leq x_1 \leq x_2 \leq$ I and $x_{n+2} = \sqrt{x_n\, x_{n+1}}$ $\forall\, n \in N$ is a Cauchy sequence.
13. If (x_n) , (y_n) be real sequences, then prove that :
    i. $\overline{\lim}\,(x_n + y_n) \leq \overline{\lim}\, x_n + \overline{\lim}\, y_n$
    ii. $\underline{\lim}\,(x_n + y_n) \geq \underline{\lim}\, x_n + \underline{\lim}\, y_n$
14. Prove that a complex sequence converges if and only if it's real part and imaginary parts converge.
15. Prove that C is complete.

16. If an infinite series is absolutely convergent, then prove that it is convergent. Is the converse true ? Give reasons in support of your answer.

17. Prove that $\sum a_n$ is convergent (or absolutely convergent) if and only if its real and imaginary parts are convergent (or absolutely convergent).

18. If $\sum a_n$ is absolutely convergent, then prove that any rearrangement of $\sum a_n$ has the same sum.

19. State and prove Abel's summation formula.

20. If $a_n, b_n, \in C \ \forall n \in N$ and

i) $S_n = a_1 + a_2 + \ldots\ldots\ldots\ldots a_n$ is bounded.

ii) $\sum | b_n - b_{n+1} |$ is convergent.

iii) $b_n \to 0$ as $n \to \infty$ then prove that $\sum a_n b_n$ is convergent.

21. Prove Leibnitz alternating series test for convergence of an infinite series.

22. If $a_n, b_n \in \mathbb{C}, \forall n \in \mathbb{C}$ and

i) $\sum a_n$ is convergent

ii) $\sum |b_n - b_{n+1}|$ is convergent

Then prove that $\sum a_n b_n$ is convergent.

23. If $\sum a_n z_0^n$ is convergent, then prove that $\sum a_n z^n$ is absolutely convergent for $|z| < |z_0|$. Hence prove that if $\sum a_n$ is convergent, then $\sum a_n z^n$ is absolutely convergent for $|z| < 1$.

24. If r is the finite radius of convergence of the power series $\sum a_n z^n$, then the series converges for $|z| < r$, diverges for $|z| > r$ and no Conclusion can be drawn on the circle of convergence. Also prove that

$$r = \overline{\lim} |a_n|^{-1/n}$$

25. If $a_n \neq 0$ for any n and $\lim |a_n/a_{n+1}|$ exists including divergence to ∞, then prove that the radius r of the circle of convergence of the power series $\sum a_n z^n$ is given by

$$r = \lim | a_n / a_{n+1} |$$

26. If the power series $\sum b_n z^n$ has radius of convergence 1 and the sequence (b_n) is monotonic and converge to zero, then prove that the power series converges $|z| = 1, z \neq 1$.

27. Prove that $|\sum_{k=0}^{n} \text{Sin } K\theta| \leq \dfrac{1}{|\text{Sin}\frac{\theta}{2}|}$ and

$|\sum_{k=1}^{n} \cos k\,\theta\ | \leq \dfrac{1}{|\text{Sin}\frac{\theta}{2}|}$ for all $n \in N, \theta = 2k\pi$, $k \in Z$

28. Discuss the convergence of

i) $\sum \frac{z^n}{(n+1)^a}$, $(a \in R)$.

ii) $\sum \frac{z^{2n}}{2n}$

iii) $\sum \frac{(-1)^n\ z^{2n}}{2n}$

iv) $\sum \frac{(-u)^{n-1}\ z^n}{n}$

v) $\sum \frac{z^n}{n!}$

29. Find the radius of convergence of each of the following power series

i) $\sum_0^\infty \frac{z^{3n}}{2^n}$ ii) $\sum_0^\infty z^{n!}$ iii) $\sum_0^\infty (n+1)\, z^n$

iv) $\sum n!\, z^{n^2}$ v) $\sum \frac{z^{2n+1}}{2n+1}$

30. If $\sum a_n$ is absolutely convergent and (b_n) is bounded, then, prove that $\sum a_n b_n$ is absolutely convergent.

31. If $\sum a_n$ converges absolutely, prove that, $\sum a_n s_n$ is absolutely convergent, where $S_n = a_1 + a_2 + \dots\dots. + a_n$.

32. If (a_n) is monotonic decreasing and $\sum a_n$ is convergent, then prove that $\sum n\,(a_n - a_{n+1})$ converges to the sum $\sum_1^\infty a_n$.

33. Test the conditional and absolute convergence of the following series.

i) $\sum_{1}^{\infty} \frac{-1^{n-1}}{n}$ ii) $\sum_{2}^{\infty}(-1)^{k}$k/logK iii) $\sum_{2}^{\infty}(-1)^{n}$ (log n)/n

iv) $\sum_{1}^{\infty} \frac{x^n}{n^a}$ v) $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \ldots\ldots\ldots$

vi) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots$

34. Examine the convergence of the following series:

i) $\sum (-1)^n a^{1/n}$, (a > 0) ii) $\frac{\text{Sinn}\theta}{\log n}$ iii) $\sum x^n/(n + x)^{2n}$, $x \in R$

iv) $\sum (n^{1/n}-1)$ Cosnx v) $\{(-1)^n/n\}$ Cos(x/n) vi) $\sum x^n/(a+n)^a$, a > 0

35. Define uniform continuity. If S is a closed and bounded subset of R such that $f : S \to R$ is continuous then prove that f is uniformly continuous on S.

36. Prove that the function $f : (0, \infty) \to R$ defined by $f(x) = 1/x$ is uniformly continuous on [a, ∞] for any a > 0 but not uniformly continuous on (0, ∞).

37. Prove that the function f, defined by $f(x) = x^2$ for all $x \in R$ is uniformly continuous on $[a, b]$ but not on $[a, \infty)$ for any $a > 0$.

38. Prove that $f(x) = Sin^2 x$ is continuous and bounded on R but not uniformly continuous.

39. If f is continuous on $[0, \infty]$ and uniformly continuous on $[k, \infty), k > 0$, then show that f is uniformly continuous on [0, ∞) .

40. Prove that a function which is uniformly continuous in a bounded interval is bounded in it.

41. If a function is continuous and bounded on a bounded interval, then is it uniformly continuous? Justify your answer.

42. Give examples of a function which is

i) Continuous but not uniformly continuous

ii) Uniformly continuous on every bounded interval and not uniformly continuous on R.

iii) Uniformly continuous but not bounded.

43. Define Riemann integral of a function f on an interval $[a, b\,]$. Prove that a function f bounded on $[a, b]$ is Riemann integrable on R if and only if for each $\epsilon > 0$, there exists a partition P of $[a, b]$ such that .

$$U\,(f, p\,) - L\,(f, p\,) < \epsilon$$

45. Prove that every monotonic bounded function on $[a, b]$ is Riemann integrable .

46. Prove that every continuous function on $[\,a,\ b\,]$ is integrable.

47. If $f \in B\ [a, b]$ be continuous over $[a, b]$ except over a finite subset of $[a, b]$, then prove that f is integrable.

48. If $f : [0,1] \rightarrow R$ be defined by

$$f(x) = \begin{cases} 1/q, \text{if } x = \text{p/q}, \text{p} \in \text{Z}, \text{q} \in \text{Z}, \text{p and q have no common factor} \\ 0, \text{ if x is irrational} \end{cases}$$

is integrable on [0,1].

49. If f is continuous on [a,b], then prove that there exists z ϵ [a,b] such that $f\ (z) = \frac{1}{b-a} \int_a^b \text{f}\,(\text{x})\, dx$.

50. If $f,\ g \in R\ [a, b]$, then prove that $R\ [a, b]$ is a linear space.

51. Let $f,\ g \in R[a, b]$, prove that

i) $f^2 \in R[a, b]$

ii) if $f(x) \le g(x), \forall\, x \in [a,\ b]$, then $\int_a^b \text{f}\,(\text{x})\ dx \le \int_a^b g(x)\, dx$

52. If f ϵR[a,b], then prove that $|f| \in R\ [a, b]$ and

$|\int_a^b \text{f(x)}\ dx| \le \int_a^b |\ \text{f}\,(\text{x})|\, \text{dx}$

Does $|f| \in R\ [a, b] \Rightarrow f\ \in R\ [a, b]$?

Give reasons in support of your answer.

53.If $a < c < b$, then prove that $f \in R[a,b]$ if and only if $f \in R[a,c]$,

and $f \in [c,b]$, and $\int_a^b \mathrm{f(x)}\, dx = \int_a^c \mathrm{f(x)}\, dx + \int_c^b \mathrm{f(x)}\, dx$

54. If $f \in R\,[a,b]$ then prove that $F(x) = \int_a^x f(t)\, dt$ is continuous on $[a,b]$

55. If $f \epsilon\, C\,[a,b)$ and $F(x) = \int_a^x f(t)\, dt,$ then prove that $F'(x) = \frac{d}{dx}\,(\int_a^x f(t)\, dt) = f(x)$ for all $x \epsilon [a,b]$.

56. If $f \epsilon\, D[a,b]$ such that $f' \epsilon\, R[a,b]$ then prove that $\int_a^b f'(x)\, dx = f(b) - f(a)$.

57. Is the function f defined by

$f(x)= \begin{cases} 1, & if\ x \epsilon \mathrm{Q} \cap [\mathrm{a,b}] \\ 0, & if\ x \epsilon (\mathrm{R-Q}) \cap [a,b] \end{cases}$ Riemann integrable on $[a,b]$?

Give reasons for your answer.

58. If $f(x) = \begin{cases} x, & x \epsilon \mathrm{Q} \\ 0, & x \epsilon (\mathrm{R-Q}), \end{cases}$ then show that $\int_a^{-b} \mathrm{f(x)}\, dx = \frac{b^2-a^2}{2}$

59. If $f \epsilon\, C\,[a,b], f(x) \geq 0$ for all $x \epsilon [a,b]$ and $\int_a^b f(x)dx = 0,$

then show that $f(x) = 0$ for all $x \epsilon [\,a,b]$

60. If $f \epsilon\, C\,[a,b]$ and $\int_a^b f\,(t)dt = 0$, then prove that there is a point $C \epsilon\, (a,b)$ such that $f(c) = 0$.

61. If $f\, \epsilon C[a,b], g \epsilon\, R[a,b]$ and $g(x) \geq 0$ for all $n \epsilon [a,\ b]$, then prove that there exists $C \epsilon\, [a,b]$ such that

$\int_a^b f(x)g(x)\, dx = f(c) \int_a^b g(x)dx.$ (First mean value theorem)

62. If $f:\ [0,1] \rightarrow R$ be defined by

$$f(x) = \begin{cases} \frac{1}{n}, & 1/(\mathrm{n}+1) < x \leq 1/n \\ 0, & \mathrm{x} = 0 \end{cases}$$

Where $n \epsilon N$, then show that f is integrable and $\int_0^1 f(x)dx = \frac{\pi^2}{6} - 1$

63. Define improper integral and show that the improper integral

$\int_1^\infty \frac{\text{sinx}}{x^a}\, dx$ converges for $a > 0$.

64. Prove that the improper integral $\int_0^\infty \frac{\text{sinx}}{x}\, dx$ is convergent but not absolutely convergent.

65. Prove that $\int_0^\infty \frac{\text{sinx}}{x}\, dx = \frac{\pi}{2}$.

66. Prove that the Gamma function defined by $\tau(\text{p}) = \int_1^\infty \text{e}^{-t}\text{t}^{\text{p-1}}\, dt$ converges for > 0.

67. Discuss the convergence of the integral $\int_0^1 \frac{\text{dx}}{x^a}$.

68. Discuss the convergence of $\int_0^\infty \text{e}^{-x}\, dx$.

69. If $f : [1, \infty) \rightarrow R$ be decreasing and non-negative, then prove be that $\sum_{k=1}^{\infty} f(k)$ converges if and only if the integral $\int_1^\infty f(t)dt$ converges.

70. Test the convergence of $\int_0^\infty \sin x^2\, dx$.

71. Test the convergence of $\int_0^\infty \frac{\text{dx}}{1+x^2}$.

72. Prove that $\tau(\frac{1}{2}) = \sqrt{\pi}$

73. Prove that i) $\tau(x + 1) = x\tau(x), 0 < x < \infty$

ii) $\tau(n + 1) = n!, n\epsilon N$

74. Prove that the integral (Beta function)

$\beta(\text{p,q}) = \int_0^1 \text{t}^{\text{p-1}} (1 - t)^{\text{q-1}} dt$ converges for $p > 0, q > 0$.

75. Discuss the convergence of $\int_1^\infty \frac{\text{logx}}{\sqrt{x}}\, dx$.

76.Discuss the convergence of $\int_1^\infty \text{e}^{x} logx\, dx$.

77. Discuss the convergence of $\int_0^{\pi/2} \log sinx\, dx$.

78. Define uniform convergence ofsequence of functions (f_n) with domain E. State and prove Cauchy criterion for uniform convergence of (f_n).

79. State and prove Weierstrass M-test for uniform convergence of a sequence of functions (f_n) with domain E.

80. Prove Dedekind's test for uniform convergence of a sequence of functions (f_n) with domain E.

81. Show that the series $\sum_1^{\infty} \frac{\sin nx}{n^a}$ is uniformly and absolutely convergent for a > 1. If 0 < a ≤ 1, then prove that the series is uniformly convergent in every interval [c, d] , where $2k\pi < c < d < 2(k + 1)\pi$, but not in any interval containing the points $2k\pi$ $(k \in Z)$

82. If $\{f_n\}$ be a sequence of functions in $R[a, b]$ converging uniformly to f, then prove that $f \in R[a, b]$ and

$$\lim_{n\to\infty} \int_a^b f_n(n)\, dx = \int_a^b f(x)\, dx.$$

83. Let $f_n \in R[a, b]$ and $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly to the sum function $f(x)$ on $[a, b]$, then prove that $\int_a^b [\sum_{n=1}^{\infty} f_n(n)]\, dn = \sum_{n=1}^{\infty} \int_a^b f_n(x)]\, dx$.

84. If (f_n) be a sequence of differentiable function with domain E^n and $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly to $f(x)$, then $f'(x) = \sum_{n=1}^{\infty} f_n'(x)$

85. Discuss the point wise convergence and uniform convergence of the following sequence of functions on the indicated sets.

i) $f_n(x) = \frac{1}{nx+1}$, $x \in [0, 1]$

ii) $f_n(x) = \frac{nx}{1+nx}$, $x \in [o, \infty)$

iii) $f_n(x)\, n = \frac{\text{Sin}\, nx}{nx}$, $x \in (0,1)$

iv) $f_n(x) = \frac{nx}{n+nx^2}$, $x \in [0,1]$.

86. Examine the pointwise and uniform convergence of the following series:

i) $\sum \frac{x^n}{1+x^n}$

ii) $\sum \frac{\sin n\theta}{n}$

iii) $\sum \frac{\cos n\theta}{n}$

iv) $\sum \frac{\sin n\theta}{n^a}$

v) $\sum \frac{\cos n\theta}{n^a}$

vi) $\sum \frac{x}{1+x^n}$

ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(+3 Degree PASS & Hons.)

Long Type

Sphere:

1. Find the equation of the sphere having the join of the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ as diameter.

2. Prove that a line in space meets a sphere in two real or coincident or imaginary points.

3. The plane $ax + by + cz + d = 0$ is a tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. Find the condition.

4. Find the equation of the sphere through the points (0, 0, 0), (-a,-b,c), (a,-b,c) and (a,b,-c) and also find its radius and centre.

5. A plane passes through a fixed point (p, q, r) and cuts the co-ordinate axes in A, B and C. Show that for all possible situations of the three points A, B and C the centre of the sphere OABC lies on the surface whose equation is $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$.

6. If a tangent plane to a sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the co-ordinate axes, prove that $a^{-2} + b^{-2} + c^{-2} = r^{-2}$.

Cone

7. Find the equation of a cone whose vertex is $C(x', y', z')$ and guiding curve is the conic given by the equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0, z = 0$.

8. Find the equation of the cone whose vertex is $C(x', y', z')$ and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$.

9. Find the condition for the general equation of second degree to represent a cone.

10. Find the equation of the Right circular cone of semivertical angle 'a' with vertex c (a,b, c) and axis $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$

11. Find the equation of the cone whose vertex is at the origin and whose guiding curve is the curve of intersection of the surface $2x^2 + 3y^2 + 4z^2 = 5$ and the plane $x + y + z = 2$.

12. Find the equation of the cone whose vertex is at the origin and whose guiding curve is the curve of intersection of the surfaces given by the equations $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$.

13. Find the equation of the enveloping cone of the sphere

$x^2 + y^2 + z^2 + 2x - 2y - 2 = 0$ with vertex at (1,1,1).

14. Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} - d = 0$

Cylinder

15. Find the equation of the cylinder whose guiding curve is a conic represented by equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ and axis represented by the equation $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

16.Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = r^2$ and generator parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

17. Find the equation of the Right Circular cylinder with radius $'r'$ and axis is the line $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$.

18. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ and whose guiding curve is the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1, z = 1$.

19. Find the enveloping cylinder of the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and the equations of whose generators are $x = y = z$.

20. Find the equation of the right circular cylinder of radius '3' whose axis is the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.

Conicoid

21. Prove that the equation of the tangent to the conicoid $ax^2 + by^2 + cz^2 = 1$ at $P\ (x', y', z')$ is $ax'x + by'y + cz'z = 1$

22. Find the condition for the plane $Ax + By + Cz = D$ to be a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$. Find the coordinates of the points of contact.

23. Prove that the points of intersection of three mutually perpendicular tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$ is a fixed sphere.

24. Find the equation of the normal at the point (x', y', z') of the conicoid

$ax^2 + by^2 + cz^2 = 1.$

25. Prove that six normals can be drawn on the central conicoid to meet a point in space.

26. The line of intersection of a pair of perpendicular tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, passes through a fixed point $(0, 0, \propto)$. Show that the line of intersection lies on the cone,

$$x^2\,(b^2 + c^2 - \propto^2) + y^2(c^2 + a - \propto^2) + (z - \propto)^2\,(a^2 + b^2) = 0.$$

27. A tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ meets the co-ordinate axes in the points A, B and C. Find the locus of the centroid of the triangle ABC.

28. If $2r$ is the distance between two parallel tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, then prove that a line through the origin and perpendicular to the planes lies on the cone

$$x^2(a^2 - r^2) + y^2\,(b^2 - r^2) + z^2\,(c^2 - r^2) = 0$$

29. Prove that the feet of the six normals drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ from any point (x', y', z') lie on the curve of intersection of the ellipsoid and the surface given by

$$\frac{a^2(b^2-c^2)x'}{x}+\frac{b^2(c^2-a^2)y'}{y}+\frac{c^2(a^2-b^2)z'}{z}=0$$

30.If the feet of the three normals from a point P to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, prove that the feet of the other three lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and P lies on the line on $a(b^2-c^2)x=b(c^2-a^2)y=c(a^2-b^2)z$.

Sphere

1. Find the equation of the sphere whose center is (-2, -1, 3) and radius 3.

2. Find the radius and center of the sphere whose equation is given by $9(x^2+y^2+z^2)-6x-12y-18z+5=0$.

3. Find the equation of the sphere through the points (0, 0, 0), (0, 1, 1), (1, 0, 1) & (1, 1, 0).

4. Find the equation of the tangent plane at (1, 1, 1) to the sphere $x^2+y^2+z^2-x-y-z=0$.

5. Find the values of 'a' such that the plane $x+y+z=a\sqrt{3}$ will touch the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$.

6. Determine the equation of the sphere passing through the point $(1,2,1)$ and the circle $x^2+y^2+z^2-9=0=z$.

7. Find the equation of the sphere passing through the circle $(x-2)^2+(y-3)^2=1, z=0$ and the point $(1,1,1)$.

8. Prove that the lines drawn from the origin touching the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ lies on the surface $d(x^2+y^2+z^2)=(ux+vy+wz)^2$.

9. Find the co-ordinates of the points where the line $\frac{x-8}{4}=\frac{y-0}{1}=\frac{z-1}{-1}=\lambda$ intersects the sphere $x^2+y^2+z^2-4x+6y-2z+5=0$.

Cone

10. Find the equation of the cone whose vertex is origin and which passes through the curve of intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$

11. Prove that the general equation of the cone of second degree passing through the axes is $fyz + gzx + hxy = 0.$

12. If a right circular cone has three mutually perpendicular generators, prove that the semi-vertical angle is $\tan^{-1}\sqrt{2}$.

13. Find the equation of a cone with vertex at the origin and axis as z-axis.

14. If the plane $2x - y + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, find the value of $'C'$.

15. Find the equation of the surface of revolution generated by revolving the curve $y + 2z + 4 = 0, x = 0$ about y axis.

16. Define Right circular cone, its vertex of generator.

17. Fill up the blanks:-

i) The tangent plane at any point of a cone passes through ___________.

ii) The tangent plane at any point P of a cone touches the cone along the generator through P. Such generator is called ___________.

iii) The vertex of the cone through which all the tangent planes pass is called ___________ point of the surface.

Cylinder:

18. Define cylinder, Right circular cylinder, Axis of the cylinder.

19. Find the equation of the Right circular cylinder whose radius is $'2'$ and axis passes through $(1, 2, 3)$ and has direction cosines proportional to $< 2, -3, 6 >$.

20. Find the equation of the Right circular cylinder whose axis is z-axis and radius $'r'$.

21. Find the equation of the cylinder which intersects the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$ and whose generators are parallel to z-axis.

Conicoid:

22. Prove that the ellipsoid is enclosed in a rectangular parallelopiped of sides $2a, 2b$ and $2c$.

23. Prove that the sections of the ellipsoid by planes parallel to the principal planes are ellipses.

24. The section of the Hyperboloid of one sheet by the plane $z = k$, is an ellipse whereas the sections by each of the planes $x = k_2$ & $y = k_3$, $|k_2| < a$ & $|k_3| < b$ are hyperbolas. Prove this.

25. Prove that the co-ordinate planes bisect all chords of the hyperboloid of two sheets normal to them.

26. Prove that the hyperboloid of two sheets has intercepts 2c along Z-axis and is intersected neither by X −axis nor by Y −axis in real points $\frac{-x^2}{a^2} + \frac{-y^2}{b^2} + \frac{z^2}{c^2} = 1$, is the equation of the hyperboloid of two sheets.

27. Identify the following surfaces only.

i. $36x^2 + 9y^2 + 4z^2 = 36$
ii. $36x^2 + 9y^2 - 4z^2 = 36$
iii. $36x^2 - 9y^2 - 4z^2 = 36$

28. Define Directorsphere, Enveloping cone of the conicoid.

29. Find the point of contact where the plane $6x + 3y - 2z = 8$ touches the conicoid $36x^2 + 9y^2 - 4z^2 = 36$.

30. Find the points of intersection of the conicoid $12x^2 - 17y^2 + 7z^2 = 7$ & the line $\frac{x+5}{-3} = \frac{y-4}{1} = \frac{z-11}{7}$

Calculus - I

(+3 1st Yr. Sc./Arts)

(Pass & Hons.)

1. Find the radius of curvature at the origin for the curve

$$x^3y - xy^3 + 2x^2y - 2xy^2 + 2y^2 - 3x^2 + 3xy - 4x = 0.$$

2. Show that, the radius of curvature for the curve $r = f(\theta)$ is $\rho = \frac{(r^2 + r_1{}^2)^{\frac{3}{2}}}{r^2 + 2r_1{}^{2)} - rr_2}$ where $r_1 = \frac{dr}{d\theta}$ & $r_2 = \frac{d^2r}{d\theta^2}$

3. For the curve $r^m = a^m \cos m\theta$, show that $\rho = \frac{a^m}{(m+1)r^{m-1}}$

4. Prove that for the cardioide $r = a\,(1 + Cos\theta), \frac{\rho^2}{r}$ is constant.

5. Find the radius of curvature for the ellipse $P^2 = a^2 \cos^2\Psi + b^2\sin^2\Psi$.

6. Prove that, the area between the curve $r = f(\theta)$ and two radii vectors $\theta = \alpha$ & $\theta = \beta$ is given by $\int_\alpha^\beta \frac{1}{2} r^2 d\theta$.

7. Find the area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

8. Find the area between the curve $y^2(a - x) = x^3$ and it's asymptote.

9.Find the area included between the parabolas $y^2 = 4ax$ & $x^2 = 4$ by.

10. Find the sum of the areas of the loops of the curve $r = a + b\,cos\theta\ (a < b)$

11. Find the area of a loop of the curve $r^2 = a^2 \cos 2\theta$.

12. Obtain Simpson's rule for approximate evaluation of the area bounded by a curve $y = f(x)$, the X-axis & two ordinates.

13. Show that the length of the arc of the curve $x = f(t)$, $y = \phi(t)$ included between two points whose parametric values are α, β is

$$\int_\alpha^\beta \sqrt{[(dx/dt)^2 + (dy/dt)]^2} dt.$$

14.Show that the length of the arc of the curve $r = f(\theta)$ included between two points whose vectorial angles are $\alpha,\ \beta$ is

$$\int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\ d\theta$$

15. Find the length of the arc of the parabola $y^2 = 4ax$ extending from the vertex to an extremity of it's latus rectum.

16. Show that, the arc of the upper half of the curve $r = a(1 - cos\theta)$ is bisected at $\theta = 2\pi/3$.

17. Find the intrinsic equation of the cycloid x = a(t + sint), y = a (1 - cost) and show that $\sigma^2 + \rho^2 = 16a^2$.

18. Find the asymptotes of $x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$.

19. Find the asymptotes of the curve $r = \frac{a}{\frac{1}{2} - cos\ \theta}$

20. Find the asymptote of the curve r sin2θ = a cos3θ.

21. If y =f(x) be a continuous function in the internal (a,b) & if the area under the curve y = f(x), X-axis and any two ordinates at x = a and x = b, be revolved about X- axis, show that the volume generated is given by

$$\int_a^b \pi\, y^2\, dx$$

22. Prove that the volume of the solid generated by revolving the curve

$y = a^3/(a^2 + x^2)$ about it's asymptote is $\pi^2 a^3/2$

23. Find the volume of the solid generated by the revolution of the cycloid

x = a(θ+ sinθ),y =a(1 - cosθ) about Y-axis.

24. The ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is divided into two parts by the line $x = \frac{a}{2}$ and the small part is rotated through four right angles about this line. Find the volume generated.

25. Find the surface obtained by revolution of the leminscate $r^2 = a^2 \, Cos2\theta$ about a tangent at the pole.

26. Find the volume formed by the revolution of the loop of the curve

$$y^2 = x^2 \frac{a-x}{a+x} \text{ about the } X\text{- axis.}$$

27.Show that the area of the surface of the solid obtained by revolving about x axis, the arc of the curve $y = f(x)$ intercepted between two points at $x = a$ & $x = b$ is

$$\int_a^b 2\pi y \frac{ds}{dx} dx.$$

28.Find the surface area of a sphere of radius a.

29. Find the surface of the solid formed by revolving the leminscate $r^2 = a^2 \cos 2\theta$ about the initial line.

30. Find the area of the surface of the frustum of a cone.

31. The curve $y^2 (a + x) = x^2 (3a - x)$ revolves about the X-axis, find the volume generated by the loop.

Objective Questions:

1. Does the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$ have a horizontal asymptote? If so, find the asymptote.

2. Find the asymptotes of the curve, $x^2 \, y^2 = c^2(x^2 + y^2)$ & show that they form the sides of a square.

3. Find the asymptotes of the curve $xy^3 + x^3 y = a^4$.

4. Find the number of points where the asymptotes of the curve $(x - y)\ (x + y)\ (x + 2y - 1) = 3x + 4y + 5$ cut it, what will be the graph of these intersecting points.

5. Find the asymptote of the hyperbolic spiral, $r\theta = a$.

6. Does the curve $y = \frac{2}{x-5}$ have a vertical asymptote? If so, find the asymptote.

7. Find the curvature of $y = ax + b$ at an arbitrary pt. (x, y).

8. Find the curvature of a circle of radius K.

9. Find the curvature of the cycloid $x = a(\theta + sin\theta),\ y = a(1 - cos\theta)$.

10. Find the radius of curvature of the curve $p^2 = ar$.

11. Compute the radius of curvature of the curve $r = acos\theta$.

12. Write the formula for radius of curvature at origin of a curve passing through the origin, where the Y-axis is a tangent there.

13. Compute the length of the curve given by $y = sinx, x \epsilon [\,0, \frac{\pi}{2}\,]$.

14. Compute the length of the arc of the parabola $y^2 = 4ax$ cut off by it's latus rectum.

15. Compute the perimeter of the cardioide $r = a(1 - cos\theta)$.

16. Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

17. Compute the area bounded by the curve $y = sinx$ and the x-axis for

$$0 \leq x \leq 2\pi$$

18. Compute the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

19. Compute the area of the cardioide $r = a(1 - cos\theta)$.

20. Compute the area of the ellipse $x = a\,cost, y = b\,sint$.

21. Find the area of the loop of the curve

$$x = a\,(1 - t^2),\ y = at\,(1 - t)^2 \text{ for } -1 \leq t \leq 1.$$

22. Find the volume of the solid of revolution of the line $y = x, x \subset [0,1)$ about x-axis.

23. Find the intrinsic equation of the curve $y = a\,cosh\,(\frac{x}{a})$.

24. Find the Cartesian equation of the curve whose intrinsic equation is $s = r\Psi$, where r is a constant.

25. Find the surface area of the solid formed by revolving the curve $r = a(1 + cos\theta)$ about the initial line.

26. Find the surface area generated if the arc AL of a parabola where, A is vertex and L is extremity of latus rectum is revolved about it's axis.

27. Find the surface of revolution of the solid generated by revolving the asteroid.

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \text{ about } x\text{-axis.}$$

28. Mention the formula for determining the area of the surface of revolution in the parametric form.

29. Mention the formula for determining the area of the surface of revolution in the polar form.

30. Mention the formula for the radius of curvature of a parametric curve.

31. Find the volume of revolution of the solid generated, by revolving the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x-axis.

ADVANCE CALCULUS

(PASS & HONS)

Multiple Choices (Tick the correct answer)

1. $x^3 + y^3 - 3axy = 0$ is a i) Explicit Function.

ii)Implicit Function.

iii)Homogeneous Function.

2. (a) If $u = f(x_1, x_2, x_3)$, $x_i = \phi(t)$ for $i = 1, 2, 3$ is

i) A function of several independent variables.

ii) An implicit function.

iii) None of these.

(b) If $u = f(x_1, x_2, x_3 \ldots\ldots x_n)$, $x_1, \ldots\ldots x_n$ are points on X-axis. Can 'u' be a function of independent variables. i) Yes.

ii) No.

3. If $u = f(x, y)$, values of 'u' define.

i) Domain of function.

ii) Range of function.

iii) Set of ordered pairs.

4. If $z = f(x, y), x^2 + Y^2 \leq a^2$, where 'a' is a constant. 'z' is

i) A closed function.

ii) Unclosed function.

iii) Open function.

5. $|x - a| \leq \delta, |y - b| \leq \delta$. Define neighbourhood of point (a, b), number δ

i) Has a infimum.

ii) Has a supremum.

iii) None of these.

6. The neighbourhood of (a, b) defined by $|x-a| \leq \delta$ and $|y-b| \leq \delta, \delta$ is

i) A rectangle.

ii) A square.

iii) Neither a square nor a rectangle.

7. Given that in every nbd of point (a, b) there is at least one point of set S.

Nbd. $|x-a| < \epsilon$ and $|y-b| < \epsilon$, where ϵ is arbitrarily small contains.

i) Infinite points.

ii) Finite points.

iii) No points.

8. The point (a, b) of Q No.7 is called. i) Point of condensation.

ii) Limit point.

iii) Accumulation point.

9. If value of $z = f(x,y)$, $y = \phi(x)$ as $x \to a, y \to b$ depends on nature of $\phi(x)$, then $f(x,y)$ has double limit at (a, b). i) No.

ii) Yes.

10. Given $\left\{x, \frac{b(x^2-a^2)}{2a(x-a)}\right\}$, and $f\left\{x, \frac{\sin b(x-a)}{(x-a)}\right\}$ tend to $f(a,b)$ as $x \to a$ does it imply double limit of 'f' exists. i) Yes

ii) No

11. When $|x-a| < \delta, \delta$ is arbitrarily small then $f(x) > f(a) + \epsilon, \epsilon$ is a small positive number, Limit of $f(x)$ exists. i) Yes

ii) No

12. Given that $\lim_{t \to 0} (1+t)^{\frac{1}{t}} \to$ 'e' then $\lim_{t \to 0} \ln\{\frac{1}{t} \ln(l+t)\} \to 0$

i) T ii) F

13. Why existence of repeated limit is not sufficient condition of the existence of double limit.

i) Repeated limit gives the value along fixed paths while double limit gives the result along all paths

ii) Converse of (i)

iii) Repeated limit reduces the function to that of a single variable.

14. If $\lim_{x\to a}\ \lim_{y\to b} f(x,y) = \lim_{y\to b}\ \lim_{x\to a} f(y,x)$,then $\lim_{\substack{x\to a\\ y\to b}} f(x,y)$ exists.

i) T ii) F

15. Given that $f(x,b)$ and $f(a,y)$ are respectively continuous at $x = a$ and $y = b$, then $f\ (x,y)$ is continuous at (a, b). i) T

ii) F

16. $f(x,y)$ is differentiable at (a, b) implies it is continuous there. i) T

ii) F

17. $f(x,y)$ is continuous at (a,b) implies $f(x,y)$ is derivable at (a,b) i) T

ii) F

18. Differentiability of $f(x,y)$ guarantees existence of i) T

$\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$ ii) F

19. (a) Converse of (Q. 18). i) T ii) F

(b).Partial derivatives exist and are bounded in the region implies continuity of the function in the region. i) T ii) F

(c). If $f(x,y)$ not differentiable then f_x, f_y discontinuous at(a,b).

i) T ii) F

20. If f_x and f_y are differentiable at (a,b), then $f_{xy} = f_{yx}$ at (a,b).

i) T ii) F

21. Given that $f(x,y) = y^n f(\frac{x^2}{y^2})$, then $xf_x + yf_y = nf(x,y)$

i)T ii) F

22. Iff $(x,y) = sin\,(\frac{x}{y} + \frac{x^2}{y^2} + \ \ldots. + \frac{x^n}{y^n})$ then $xf_x + yf_y = 0$

i) T ii) F

23. If $f(x,y) = ln\,(\frac{x^2}{y}) + ln\,\left(\frac{y^2}{x}\right)$ then $xf_x + yf_x = f(x,y)$.

i) T ii) F

24. The distinction between Jacobian and derivative is that between a determinant and its element. i) T ii) F

25. Young's and Schwarz's theorems are necessary conditions for $f_{xy} = f_{yx}$

i) T ii) F

26. Stationary points of a function are same as its extreme points .

i) T ii) F

27. If each of $f_{x_1} . f_{x_2} \ldots\ldots f_{x_n}$ is zero at (a_1 , a_2 ,..., a_n)

f (a_1 , a_2 ,...a_n) is an extreme valueof the function. i) T ii) F

28. Double integral in general measures? i) Length

ii) Area

iii) Volume

29. Integrals $\int\int$ f(x,y)ds and ds an element of surface $\int\int$ f(x,y) dx dy are same. i) T ii) F

30. Changing the order of integration changes its value i) T ii) F

Vector Calculus:

31. $\frac{d}{dt}$ $(\bar{F}\text{x}\bar{G}) = \frac{d}{dt}$ $(\bar{F} \text{ x } \bar{G}) + \frac{d}{dt}$ $(\bar{G} \text{ x } \bar{F})$, $\bar{F}$, $\bar{G}$ are vector point functions of the variable 't'. i) T ii) F

32. If $\bar{F}$is a constant vector function of t; $\frac{d\bar{F}}{dt}$ is normal to $\bar{F}$. i) T

ii) F

33. Operator $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$ is invariant w.r.t. coordinate system.

i) T ii) F

34. $\nabla\phi(x,y,z)$, where $\phi(x,y,z)$ is a scalar point function is a vector function.
i) T ii) F

35. If $\frac{\nabla\phi\,(x,y,z)}{|\nabla\phi|}$ = (f_1, f_2, f_3) , then f_1, f_2, f_3 measure the direction cosine of the normal to the surface $\phi(x, y, z) = C$. i) T ii) F

36. Directional derivative is a vector quantity. i) T ii) F

37. Directional derivative of $\phi(x, y) = c$ along the Z-axis is gradient of $\phi(x, y)$. i) T ii) F

38. If $(\vec{r}-\vec{r}_0)\,\nabla\phi = (\vec{r}-\vec{r}_0) \times \nabla\phi$; the locus of $\vec{r}$ is a straight line through $\vec{r}_0$.

i) T ii) F

39. Sum of the directional derivatives along X, Y and Z axes measures the gradient. i) T ii) F

40. Directional derivative of $\phi(x,y,z) = c$ is greatest along the tangent.

i) T ii) F

41. Div($\vec{F}$) is a vector quantity. i) T ii) F

42. Div [curl (grad f)] = curl [grad(div f)] i) T ii) F

43. Directional derivative has the greatest value along the gradient. i) T

ii) F

44. Is it possible to have | directional derivative | = | Gradient | i) T ii) F

45. Curl measures rotational tendency.

46. Divergence measures concentration of fluid at a point. i) T ii) F

47. $\int_{AB} y\,dx = y^X AB$, where AB is a length along X-axis. i) T ii) F

48. $\oint \nabla\emptyset . d\vec{r} = 0$ i) T ii) F

49. If $\vec{F}$ is solenoidal $\int_{0AB0} \vec{F} . d\vec{r} = o$ i) T ii) F

50. If curl $\vec{F} = 0$, then $\oint \vec{F} . d\vec{r} = 0$ i) T ii) F

Long Questions:

Vector Calculus:-

1. i) Prove that every derivable function $\vec{F}$ is continuous.

ii) If $\vec{f}$(t) = (f_1 (t), f_2 (t), f_3 (t)) is derivable, so are f_1 (t), f_2 (t), f_3 (t). Prove this.

2. Show that $\vec{r} = \vec{a}e^{kt} + \vec{b}e^{lt}$ is a solution of $\frac{d^2\vec{r}}{dt^2} + p\frac{d\vec{r}}{dt} + q\vec{r} = 0$, where P and q are constants and k, l are distinct roots of $m^2 + pm + q = 0$; $\vec{a}, \vec{b}$ are arbitrary vectors.

3. i) Given that $\hat{r}$ is a unit vector, show that

$$|\hat{r} \times \frac{d\hat{r}}{dt}| = |\frac{d\hat{r}}{dt}|$$

ii) Find the derivative of $r^n \vec{r}$

4. Obtain the first and the second derivative of

i) $\vec{r} \times (\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2})$; ii) $[\vec{r}, \frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}]$

5. i) If $\vec{r} \times \frac{d^2\vec{r}}{dt^2} = 0$, find $\vec{r} \times \frac{d\vec{r}}{dt}$

ii) Find out $\vec{r}$ so that $\frac{d^2\vec{r}}{dt^2} = \vec{a}t + \vec{b}$. Given that $\vec{r}, \frac{d\vec{r}}{dt}$ vanish at $t = 0$.

6.Integrate:

i) $\int (\vec{r}.\frac{d\vec{s}}{dt} + \vec{s}.\frac{d\vec{r}}{dt})$; ii) $\int 2 (\frac{d\vec{r}}{dt}.\frac{d^2\vec{r}}{dt^2})\, dt$;

iii) $\int (\frac{1}{r}\frac{d\vec{r}}{dt} - \frac{dr}{dt}\frac{\vec{r}}{r^2})\, dt$ iv) If $\frac{d\vec{f}}{dt} = \vec{F}(t)$, for all $t \in [t_1, t_2]$

then evaluate $\int_{t1}^{t2} \vec{F}(t)dt$.

7. Evaluate $\int_2^3 (\vec{r}.\frac{d\vec{r}}{dt})$, given that $\vec{r}(t) = \begin{cases} 2\hat{\imath} - \hat{\jmath} + 2\hat{k} \text{ at } t = 2 \\ 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k} \text{ at } t = 3 \end{cases}$

8. Acceleration of a moving particle is given by the expression 12cos2t $\hat{\imath}$ - 8sin2t $\hat{\jmath}$ - 16t $\hat{k}$. Obtain expression for velocity and displacement if at t = 0 both velocity and displacement vanish.

9. If $\phi(x,y,z) = xy^2z$ and $\vec{u} = xz\hat{\imath} - xy\hat{\jmath} + yz^2\hat{k}$, find the value of $\frac{\partial^3 \phi \vec{u}}{\partial x^2 \partial z}$ at (2,-1, 1).

10. A particle moves along the curve $x=e^{-t}$, y=2 cos3t,z=2 sin3t, where t denotes time. Determine it's velocity and acceleration at $t = 0$.

11. If $\frac{d\vec{a}}{ds} = (\vec{r} \times \vec{a})$ and $\frac{d\vec{b}}{ds} = (\vec{r} \times \vec{b})$ then show that

$$\frac{d(\vec{a} \times \vec{b})}{ds} = \vec{r} \times (\vec{a} \times \vec{b}).$$

12. Find the equation of tangent line at (1, 0, 0) to the curve of intersection of $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$.

13. Determine the directional derivative of the function $f(x,y,z) = x^2 - y^2 + 2z^2$ at the point p(1,2,3) along the direction of the line $P\vec{Q}$, where Q is the point (5, 0, 4).

14. Obtain the derivative of $f(x,y,z) = x^2\, y^3\, z^4$ along the direction that makes equal angle with the coordinate axes.

15. i) Find grade f, where $f = x^3 - y^3 + xz^2$ at (1, -1, 2)

ii) Find ∇ r^n where $\vec{r} = (x,y,z)$.

16. i) Find the greatest rate of increase of $u = xyz^2$ at (1,0,3)

ii) Find out the unit normal to the surface $x^2y + 2xz = 4$ at (2,-2,3).

17. i) Show that $\nabla(\vec{r}.\vec{a}) = \vec{a}$, $\vec{a}$ being a constant vector.

ii) Find the equation of the tangent plane and normal line to the surface

$xyz = 4$ at the point (1,2,2).

18. Show that i) $\nabla^2[\nabla.\frac{\vec{r}}{r^2}] = \frac{2}{r^2}$;

ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ and hence deduce that if $\nabla^2 f(r) = 0$ then $f(r) + c_1 + \frac{c_2}{r}$

19. Show that $\vec{u} = r^n \vec{r}$ is solenoidal when $n + 3 = 0$ and is irrational for other values of n. [Obtain the following results: (Q. Nos. 20,21,22)]

20. i) grad $(\phi\Psi) = \phi$ grad $\Psi + \Psi$ grad ϕ

ii) grad $(\vec{f}.\vec{g}) = \vec{f}$x curl $\vec{g}$ +$\vec{g}$x curl $\vec{f}$+ (f.∇) $\vec{g}$ + ($\vec{g}$.∇) $\vec{f}$.

21. i) div $(\phi\vec{f}) = \phi$ div $\vec{f} + \vec{f}$. gradϕ;

ii) div $(\vec{f}$x$\vec{g})$= $\vec{g}$ curl $\vec{f}$ - $\vec{f}$. curl$\vec{g}$

22. i) curl $(\phi\vec{f})$ = grad ϕ x $\vec{f}$+ ϕ curl $\vec{f}$;

ii) curl ($\vec{f}$x $\vec{g}$) =$\vec{f}$ div $\vec{g}$ - $\vec{g}$ div $\vec{f}$ + ($\vec{g}.\nabla$) $\vec{f}$- ($\vec{f}$. ∇) $\vec{g}$.

show that (Q Nos.23, 24)

23. i) div grad $\phi = \nabla^2\phi$; ii) curl grad $\phi = 0$; div curl $\vec{f} = 0$;

24. curlcurl$\vec{f}$ = grad div $\vec{f} - \nabla^2\vec{f}$

25. If $\vec{a}$ is a constant vector, prove that

Curl $\vec{a}$ x $\frac{\vec{r}}{r^3} = (\frac{-\vec{a}}{r^3} + \frac{3\,\vec{a}.\vec{r}}{r^5})\,\vec{r}$.

26. Prove that $(\vec{v}.\nabla)\,\vec{v} = \frac{1}{2}\,\vec{v}\,\nabla\,v^2 - \vec{v}$ x (∇x $\vec{v}$).

27. Show that i) Curl { ($\vec{r}$x$\vec{a}$) x ($\vec{r}$x$\vec{b}$) } = ($\vec{a}$x$\vec{b}$)x$\vec{r}$

ii) Div{ ($\vec{r}$x$\vec{a}$) . ($\vec{r}$x$\vec{b}$) } = 4 $\vec{a}$.($\vec{b}$x$\vec{r}$) ;

where $\vec{a}$, $\vec{b}$ are constant vectors.

28. If $\vec{f}$ is solenoidal, prove that Curl curl curl curl $\vec{f} = \nabla^2\nabla^2\vec{f} = \nabla^4\vec{f}$.

29. i) State Stoke's theorem on integral transform.

ii) State Gauss's theorem on integral transform.

30. Integrate $\vec{F} = x^2\hat{\imath} - xy\hat{\jmath}$ from 0(0, 0) to P(1,1)

i) Along the line OP

ii) Along the parabola $y^2 = x$

iii) Along the X-axis from 0 to 1 , then along $x = 1$ from 0 to 1. Does $\vec{F}$ represent a conservative field ?

31. Find out the circulation of $\vec{F}$ = (y,z,x) round the curve $x^2 + y^2 = 1$, z = 0.

32. Given $\vec{F}$ = (y,x-2xz, xy) evaluate $\int_s \nabla \times \vec{F} \cdot \hat{n}ds$, where S is the surface $x^2 + y^2 + z^2 = a$ above the xy-plane.

33.i) Show that $\int_v dv/2r^2 = \int_s (\vec{r} \cdot \vec{n} / r^2)ds$

ii) $\int_s \text{curl}\, \vec{F} \cdot \vec{n}ds = 0$ for any closed surface S.

34. Compute aer $\oint (ax^2 + by^2 + cz^2)$ ds, over the sphere $X^2 + y^2 + z^2 = 1$ using divergence theorem.

35. Verify Stoke's theorem for $\vec{F} = (x^2, xy)$ integrated round the square in the Z-plane $z = 0$ whose sides are the lines $x = 0,\ y = 0$,

$x = a, y = b$.

ADVANCED CALCULUS

(Functions of several variables)

1. Obtain the limit of $\lim_{\substack{x \to 0 \\ y \to 0}} xy - \frac{x^2 - y^2}{x^2 + y^2}$

2.Show that the limit as $(x, y) \to (0,0)$ does not exist in each of the following cases :

i) $\frac{xy}{x^2 + y^2}$, ii) $\frac{x^2y^2}{x^2y^2 + (x^2 - y^2)^2}$ iii) $\frac{x^3 + y^3}{x - y}$

3. Deduce ab initio the following results:

i) $\lim_{\substack{x \to 1 \\ y \to 2}} 3xy = 6$, ii) $\lim_{(x,y) \to (1,2)} (x^2 + 2y) = 5$

4. Show that for the function

$$f(x,y) = \begin{cases} x \operatorname{Sin}\left(\frac{1}{y}\right) + y \operatorname{Sin}\left(\frac{1}{x}\right), & xy \neq 0 \\ 0, & xy = 0 \end{cases},$$

limit exists at the origin but repeated limits do not exist.

5. Show that in the following cases the repeated limits exist but not the double limit.

i) $f(x, y) = \frac{x^2y^2}{x^4 + y^4 - x^2y^2}$,

ii) $f(x) = \begin{cases} (x^2 - y^2)/(x^2 + y^2), & x \neq y \\ 0, & x = y \end{cases}$

6. Show that the following functions are discontinuous at the origin;

i) $f(x, y) = \begin{cases} \frac{1}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

ii) $f(x, y) = \begin{cases} \frac{x^2y^2}{x^4 + y^4} & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

7. Discuss the following functions for continuity at (0, 0);

i) $f(x,y) = \begin{cases} \frac{x^2y}{x^3+y^3}, & x^2+y^2 \neq 0 \\ 0, & x=y=0 \end{cases}$.

ii) $f(x,y) = \begin{cases} 0, & (x\ y) \neq (2y,y) \\ exp\left\{\frac{|x-2y|}{x^2-4xy+4y^2}\right\}, & (x\ y) = (2y,y) \end{cases}$

8. How can you define the following functions at (0, 0) so that they may become continuous there.

i) $f(x,y) = \sin\frac{x}{y}$, ii) $f(x,y) = \frac{x^3+y^3}{x^2+y^2}$

9. For the function $f(x,y) = \frac{xy\,(x^2-y^2)}{x^2+y^2}$ $, x^2+y^2 = 0$ and $f(0,0) = 0$, show that i) $f_x(x,0) = f_y(0,y)$, ii) $f_x(0,y) = -y$, iii) $f_y(x,0) = x$

10. State and prove the "Mean Value Theorem" for a function of two independent variables.

11. If f(a, b) be a point of the domain of definition of a function f(x, y), such that

i) f_x is continuous at (a, b), ii) f_y exists at (a, b)

then prove that $f(x,y)$ is differentiable at (a, b).

12. If f_x and f_y are both differentiable at the point (a, b) in the domain of the function $f(x, y)$, then show that $f_{xy}(a, b) = f_{yx}(a, b)$

13. If f_y exists in a Certain neighbourhood of a point (a, b) of the domain of definition of $f(x,y)$ and f_{yx} is continuous at (a, b), then prove that $f_{xy}(a, b)$ exists and is equal to $f_{yx}(a, b)$.

14.Show that for the function

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & x=y=0 \end{cases}.$$

f_{xy} (0,0) = f_{yx} (0,0) even though conditions of Schwarz's and Young's theorem are not satisfied.

15. Show that $(\frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 = (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2$;

Where $x = u \cos a - v \sin a, y = u \sin a + v \cos a, a$ is constant.

16. If $z = a \tan^{-1} \frac{y}{x}$, show that $(1 + q^2)$ r - 2pqs + $(1 + k^2)$t=0

and $\frac{(rt - s^2)}{(1 + p^2 + q^2)^2} = \frac{-a^2}{(x^2 + y^2 + a^2)^2}$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, s = \frac{\partial^2 z}{\partial xoy}, r = \frac{\partial^2 z}{\partial x^2}, t = \frac{\partial^2 z}{\partial y^2}$

18. If z =$f(\frac{ny - mz}{nx - Lz})$, prove that $(nx - lz)\frac{\partial z}{\partial x} + (ny - mz)\frac{\partial z}{\partial y} = 0$.

19. If $u = \phi(x + at) + \Psi(x - at)$, then show that $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

20. If $= f(y - z, z - x, x - y)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

21. If u= x $\phi(\frac{y}{x}) + \Psi(\frac{y}{x})$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial xoy} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$

22. If u = $\text{Sin}^{-1}(\frac{x}{y}) + \tan^{-1}(\frac{y}{x})$, show that $xu_x + yu_y = 0$.

23. If u = $\tan^{-1}(\frac{(x^3 + y^3)}{x - y})$, determine the value of $x^2u_{xx} + 2xy\, u_{xy} + y^2u_{yy}$

24. Expand $e^x\tan^{-1}y$ about (1,1) up to second degree in (x-1) and (y-1).

25. If x = r sinθ cosϕ, y = r sinθ sinϕ, z = r cosθ, then show that

$$\frac{\partial(x,Y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta.$$

26. Show that $u = x + y - z, v = x - y + z, w = x^2 + y^2 + z^2 - 2yz$ are independent of one another. Obtain a relation connecting u, v, w.

27. Show that $ax^2 + 2hxy + by^2$ and $Ax^2 + 2Hxy + By^2$ are independent unless $\frac{a}{A} = \frac{b}{B} = \frac{h}{H}$.

28. Define "Saddle Points" of a function $f(x, y)$. Test the function $x^2 - xy + y^4$ for relative maxima, relative minima and saddle points.

29. Prove that the volume of greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8\,abc}{3\sqrt{3}}$.

30.i) If $2x + 3y + 4z = a$, the maximum value of $x^2 y^3 z^4$ is $\left(\frac{a}{9}\right)^9$.

ii) If $xyz = abc$, the minimum value of $bcx + cay + abz$ is $3abc$.

31. Show that the point on the sphere $x^2 + y^2 + z^2 = 1$, which is further from (2,1,3) is $(-\frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}})$.

32. Show that the maximum and minimum distance from the origin to the curve of intersection defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and, $lx + my + nz = 0$ can be obtained by solving for 'a' the equation

$$\frac{a^2 l^2}{a^2 - d^2} + \frac{b^2 m^2}{b^2 - d^2} + \frac{c^2 n^2}{c^2 - d^2} = 0.$$

33. Find the value of $\int_c (x + y^2)dx + (x^2 - y)\,dy$ taken in the clock sense along the closed curve C formed by $y^3 = x^2$ and the chord joining (0,0) with (1,1)

34. Show that $\int_C \frac{y^2\,dx - x^2\,dy}{x^2 + y^2} = -\frac{4a}{3}$, C is the semi circle

$x = a \cos t$, $y = a \sin t$ from $t = 0$ to $t = \pi$.

Computer Programming

1. a) Write the names of main functional units of a digital computer.

b) Mention the elements of Fortran IV character set.

c) Mention the names of library functions of Fortran IV language.

d) Write the hierarchy of arithmetic operators.

2. Write the corresponding Fortran expression for the following mathematical ones.

a) $f = tan^3\left(\frac{ax}{\left(\frac{n+1}{2}\right)a}\right) + \frac{logx}{a^2 - B^2}$

b) $\frac{1}{2r^2(\theta - Sin\theta)} + \frac{4}{3}\ \pi r^3$

c) $\frac{1}{2\,a^2x^4} - \left(\frac{1}{a^2}\right) tan^{-1}\frac{x}{a} - e^{-(\frac{x^2}{x^2 + a^2})}$

d) $\sigma = \sqrt{f.\frac{(x - \bar{x})^2}{n} - f.\frac{x}{n^2}}$

3 a) Why are the following unacceptable as Fortran real constants?

i) 16.E 128 ii) 4E-2 iii) 3,246.6

b) Which of the following are unacceptable names for the integer variables and why ? J12, J123478, LOK2, AJ, M4.2

4. Distinguish between (through examples)

a) Logical constants and literal constants
b) Executable statements and non - executable statements
c) Arithmetic IF statement and Logical IF statement
d) Input statements and Output statements
e) F-format Code and E-format code.

5. Write short notes on:

a) Programming languages. b) Job cards

c) DIMENSION statement. d) DATA statement.

e) I-Format Code. f) H-Format Code.

6. a) Point out the errors in the following program segments :

i) DIMENSION Y(50)

```
      J = 55
y(J) = 2 * J+1
```

ii) DIMENSION M (10)

```
      I = J
M (I-J) = I* J
```

iii) I + J = 55

```
GOTO, I + J
```

iv) DO I = 3,1,-1

b) Point out the errors in the following program giving reasons :

```
C     SUM OF A(1). A(2) , A (3), A(4), A(5),
      READ (6,10),A
10    FORMAT (5 I 7)
      SUM = 0
      DO 11      J = 5.1, -1
      SUM = SUM + A(J)
11    CONTINUE
      WRITE (5,12) SUM
12    FORMAT ( I20)
      END
      STOP
```

7.(a) Suppose three data cards are punched as follows:

1st card:	11.1	22.2	333
2nd card:	444	555	66. 66
3rd card :	7.777	8.88	

Find the values assigned to the variables if the following read statements are executed:-

i) READ,A,B,J,K

ii) READ, X,Y

b) Following is the design of a data card:

1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0

- 3 .567 . 2E5 b 87 E b 688 . 3

Write the values of the variables which will be read by the following READ statements:

i)
```
      READ 5, A,B,C,D
    5 FORMAT ( E5.0, 2F6.0, F3.0)
```

ii)
```
      READ 50, I,J,K,L
   50 FORMAT ( 3X,I3,3X,I3,2X,I4,1X,I1)
```

8.a) What is the final value of X in the following program :

```
X = 2.54
X = (X + .05) * 10
I = X
X = I
X = X /10
```

b) Write a program segment to complete the following:

$$f(x) = \begin{cases} 2x, & x > 0 \\ 1, & x = 0 \\ x2, & x < 0 \end{cases}$$

9. (a).Write the grammatical rules for DATA statements, explaining them briefly.

(b) What are the usefulness of IMPLICIT type specification statements? Describe the rules governing them.

10. Discuss the need and rules of a DIMENSION statement in Fortran IV language. Give an example of a program in which a two dimensional array occurs and explain the program.

11. a) Draw a flow chart to sum the series $\sum_{n=0}^{10} \frac{x^n}{n!}$.

b) Draw a flow chart which determines the largest of 50 given numbers.

c) Draw a flowchart to find the roots of the eqn. $ax^2 + bx + c = 0$, $(a \neq 0)$.

d) Draw a flow chart to sum of the series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$ for x = 0.5, correct to five decimal places .

12. a) Write a Fortran program which inputs three numbers a, b and c which are the three sides of a triangle and computes and prints the area AREA and PERI, the perimeter of the triangle.

b) A is an one-dimensional array with 500 elements. Write a program which evaluates the arithmetic mean (AM) and standard deviation (SD) of these elements and prints AM & SD.

c) Draw the flow chart and write a program which reads k and determines $k\,!\ (k > 0)$.

d) Write a Fortran Program which arranges an array X consisting of 70 elements in descending order.

e) Write a program which evaluates $y = x^2 - 4x + 6$ for -5(0.5)5 and prints x and y.

f) Write a program to evaluate the following integral by using Simpson's 1/3rd rule:

$$\int_1^3 \frac{x^2+1}{(x^3+x+1)^{\frac{1}{2}}}dx$$

DIFFERENTIAL EQUATIONS

Objective questions:-

1. What is the difference between an ordinary differential equation and partial differential equation ?

2. What is a linear differential equation ?

3. What is the difference between the critical value problem and boundary value problem ?

4. What is a homogeneous differential equation ?

5. What is an exact differential equation ?

6. What is the condition such that a differential equation is exact ?

7. What is an integrating factor ?

8. When a differential equation is in the Clairaut 's form ?

9. What is the auxiliary equation in a differential equation ?

10. Find sin $\frac{ax}{D^2 + a^2}$

11. What is the value of $\frac{e^{ax}}{D^2 + a^2}$?

12. What is the value of $\frac{\sin 3x}{D^2 + 4}$?

13. What is the value of $\frac{e^x \sin x}{D^3 + 1}$?

14. What is L (Cosat) ?

15. What is L (Sinat) ?

16. What is Laplace transform of 1 ?

17. State the shifting theorem.

18. State the unit step function.

19. What is unit Impulse function ?

20. What is a singular solution in a differential equation?

21. Write down the differential equation whose solution is

i) $y = cx + c^2$

ii) $y = e^{cx}$

22. Write the differential equation whose solution is a family of parabolas with vertices at origin and foci on X -axis.

23. Write the differential equation whose solution is a family of circles of variable radii with centre at the origin.

24. What is the solution of $\frac{dy}{dx} = \ln x$?

25. What is the solution of $y' + 2y = 0$?

26. What is the solution of $x + yy' = \phi$

27. What is the solution of $\frac{d^2y}{dx^2} + y = \sin x$?

28. What is the Bernoulli's Equation?

29. What is the Riccati Equation ?

30. Find $\frac{x^2}{D^2+1}$.

Write TRUE or FALSE

31. $Y = |x|$ Satisfies $\frac{dy}{dx} = 1$ in $(-\infty, 0)$

32. $y = |x|$ satisfies $\frac{dy}{dx} = -1$ in $(-\infty, 0)$

33. $y = |x|$ is the solution of $y' = 1$ in $(-\infty, \infty)$

34. $y = \sqrt{-(1 + x^2)}$ is a solution of $x + yy' = 0$.

35. $x^2 + y^2 = 0$ is an implicit solution of $x + yy' = 0$. Select the correct answer from the given choices for each of the following question.

36. The first order differential equation xy' = 1 has

a. No solution in (-3,3)
b. Has solution y = lnx + con ($-\infty, \infty$)
c. Has solution y = ln|x| + c on ($-\infty, \infty$)
d. None of these.

37. The general solution of $\frac{dy}{dx} = cx$ is

a. $y = \frac{cx^2}{2} - c$
b. $y = \frac{cx^2}{2} + c$
c. $y = \frac{cx^2}{2} + c^2$
d. $y = \frac{cx^2}{2} + c_1$

38. The solution of the initial value problem $\frac{dy}{dx} + y = x$, Y(o) = 1 is

a) $Y = x + 2e^{-x} - 1$
b) $y = x + 1$
c) $y = xe^x - e^x + 2$
d) $y = x - 1 + ce^{-x}$

39. The solution of the initial value problem

$x\frac{dy}{dx} - 2y = x^3 e^x, y(1) = 0$ is

a) $y = e^{-x}(\frac{x^2e^{-2x}}{2} - 1)$
b) $y = e^x(\frac{x^3}{3} - 1)$
c) $y = x^2(e^x - e)$
d) $y = \frac{x^4e^x - e}{x^2}$

40. The solution to the initial value problem.

$\frac{dy}{dx} + 5y = 3e^x$, Y(0) = 1 is

a) $y = \frac{1}{2}(e^x + e^{-2x})$
b) $y = \frac{1}{2}(e^x + e^{-3x})$

c) $y = ½ (e^{x} + e^{-4x})$
d) $y = ½ (e^{x} + e^{-5x})$

41. The general solution of $y = xp + p^2$ is

a) $y = cx + c$
b) $y = cx + 2c$
c) $y = cx + c^2$
d) $y = cx + c^3$

42. The general solution of $y''' - y' = 0$ is

a) $y = c + ce^{x} + ce^{-x}$
b) $y = c + c_1e^{x} + c_1 e^{-x}$
c) $y = c + c_1e^{x} + c_2 e^{-x}$
d) $y = c + ce^{x} + c_1 e\text{-}^{x}$

43 The general solution of y" + 8y = 0, is

a) $y = c_1 \cos 2x + c_2 \sin 2x$

b) $y = c_1 \cos \sqrt{2x} + c_2 \sin \sqrt{2x}$

c) $y = c_1 \cos 2\sqrt{2x} + c_2 \sin 2\sqrt{2x}$

d) $y = c_1 \cos 4x + c_2 \sin 4x$

44. The solution of the initial value problem y" - 3y' - 4y = o , y (o) = 1 , y'(o) = o is

a) $y = \frac{1}{5}(e^{4x} + e^{-4x})$ b) $y = \frac{1}{5}(e^{4x} + e^{-3x})$

c) $y = \frac{1}{5}(e^{4x} + e^{-2x})$ d) $y = \frac{1}{5}(e^{4x} + e^{-x})$

45. Laplace transform of acos2t is

a) $\frac{a}{p^2+4}$ b) $\frac{ap}{p^2+4}$

c) $\frac{p}{p^2+4}$ d) $\frac{1}{p^2+4}$

46. Laplace transform of e^{at} is

a) $\frac{1}{p+a}$

b) $\frac{1}{p-a}$

c) $\frac{1}{p^2+a^2}$

d) $\frac{1}{p^2-a^2}$

47. Laplace transform of t is

a) $\frac{1}{p}$

b) $\frac{1}{p^2}$

c) $\frac{1}{p^3}$

d) $\frac{1}{p^4}$

48. The inverse transform of $\frac{a}{p^2+a^2}$ is

a) cos at

b) sin at

c) tan at

d) cot at

49. The Laplace transform of $\frac{\sin wt}{t}$ is

a) $\sin^{-1}\frac{w}{p}$

b) $\cos^{-1}\frac{w}{p}$

c) $\tan^{-1}\frac{w}{p}$

d) $\cot^{-1}\frac{w}{p}$

50 Laplace transform of t sin 3t is

a) $\frac{2p}{(p^2+q)^2}$

b) $\frac{4p}{(p^2+q)^2}$

c) $\frac{6p}{(p^2+q)^2}$

d) $\frac{8p}{(p^2+q)^2}$

Long Questions

Solve the following equations.

1. $2xy\,dy - (y^2 - x^2)\,dx = 0$

2. $(2x + y + 3)\frac{dy}{dx} = x + 2y + 3$

3. $(\sin x \sin y - x e^y)\,dy = (e^y + \cos x \cos y)\,dx$

4. $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$

5. $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$

6. $x \log x \frac{dy}{dx} + y = 2 \log x$

7. $(1+y^2)\,dx = (\tan^{-1} y - x)\,dy$

8. $x\frac{dy}{dx} + y \log y = xye^x$

9. $\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$

10. $3y^2\frac{dy}{dx} + xy^3 = x$

11. $xy\,p^2 + p(3x^2 - 2y^2) - 6xy = 0$

12. $xp^2 - 2yp + ax = 0$

13. $y^2 \log y = xy\,p + p^2$

14. $x^2 = a^2(1 + P^2)$

15. $xy^2(p^2 + 2) = 2py^3 + x^3$

16. $x^2(y - px) = p^2y$

17. $x\,p^2 + (x - y)p - x = 0$

18. $(D^2 + 1)y = \sin x - \cos 2x$

19. $(D^5 - D)y = 12e^x + 8\sin x - 2x$

20. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$

Solve the following initial value problem

21. $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y, y(1) = 0$

22. $x^2 \frac{dy}{dx} + 2xy - x + 1 = 0, y(1) = 0$

23. $\frac{dy}{dx} + \frac{1}{x} y = \frac{\sin x}{x}, y(0) = 0$

24. $(2x + e^x \sin y)\, dx + e^x \cos y\, dy = 0, y(0) = \frac{11}{2}$

25. $(3x^2 y + 8xy^2)\, dx + (x^3 + 8x^2 y + 12y^2)\, dy = 0, y(2) = 0$

26. Find the Laplace transform of

a) $e^{-t} \cos^2 t$, b) $e^{-t} \sin wt$

27. Find the inverse transform of

a. $\frac{p+2}{(p+2)^2 + w^2}$

b. $\frac{p+3}{p^2 + 2p + 10}$

28.If $\bar{f}(p)$ is the transform of f(t), then prove that

$\frac{d^n \bar{f}(P)}{dp^n} = L\{(-t)^n + f(t)\}$

29. Find the Laplace transform of

a) $te^{-t} \cos 2t$ b) $-\frac{t}{a^2}(1 - \cos at)$

30. Solve the differential equation

$y'' + 2y' + 5y = 8 \sin t + 4 \cos t, y(0) = 1, y'(0) = 3$

Differential Geometry

1. Establish Serret - Frenet formulae.

2. Show that the ratio of curvature and torsion of the circular helix

$$\vec{r} = a\,(\cos\theta, \sin\theta, \theta\cot\beta) \text{ is constant.}$$

3. For the curve

$$x = a(3u - u^3),\ y = 3au^2,\ z = a(3u + u^3),$$
$$k = \tau = \frac{1}{3a\,(1 + u^2)^2}$$

4. If the tangent and the binormal at a point of a curve make angles θ&ϕ respectively with a fixed direction, show that

$$\frac{\sin\theta.\,d\theta}{\sin\phi.\,d\phi} = -\frac{k}{\tau}$$

5.For the curve $x = 4a\cos^3 u$, $y = 4a\sin^3 u$, $z = 3c\cos 2u$

prove that $k = \dfrac{a}{6\,(a^2 + c^2)\sin 2u}$.

6. Find the curvature and torsion of the curve

$$x = a\,(u - \sin u),\ y = a(1 - \cos u),\ z = bu.$$

7. Prove that for any curve

$$[\hat{t}'\hat{t}''\ \hat{t}'''] = k^5 \frac{d\frac{\tau}{k}}{ds}.\text{and}$$

$$[\hat{b}'\ \hat{b}''\ \hat{b}'''] = \tau^5 \frac{d\frac{k}{\tau}}{ds}.$$

8. Find the curvature and torsion of an involute.

9. Prove that, in order that the principal normals of a curve be binormals of another, the relation $a(k^2 + \tau^2) = k$ must hold,where 'a' is a constant.

10. Find the equations of tangent plane and normal at a point (x,y,z) to the surface F(x,y,z) = 0.

11. Prove that the tangent plane to the surface $xyz = a^3$, and the coordinate planes, bound a tetrahedron of constant volume.

12. Show that the sum of the squares of the intercepts on the coordinate axes made by the tangent plane to the surface.

$$x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3} \text{ is constant.}$$

13. The normal at a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ meets the coordinate planes in G_1, G_2, G_3. Prove that the ratios $PG_1 : PG_2 : PG_3$ are constant.

14. Spheres of constant radius b have their centres on the fixed circle $x^2 + y^2 = a^2, z = 0$. Prove that their envelope is the surface

$$(x^2 + y^2 + z^2 + a^2 - b^2)^2 = 4a^2 (x^2 + y^2).$$

15. Find the envelope and the edge of regression of the family of ellipsoids $c^2(\frac{x^2}{a^2}+\frac{y^2}{b^2}) + \frac{z^2}{c^2} = 1$, where *c* is the parameter.

16. Find the condition that the surface given by the equation $z = f(x,y)$ will be a developable.

17. Show that the surface $xy = (z - c)^2$ is a developable.

18. For the surface of revolution $\vec{r} = [u \cos v, u \sin v, f(u)]$, find the first order magnitudes and further show that $(ds)^2 = (1+ f'^2)\, du^2 + u^2\, dv^2$.

19. If Ψ be the angle between the two directions given by $P du^2 + Q\, du dv + r\, dv^2$, then show that $\tan \Psi = \frac{H\sqrt{Q^2 - 4PR}}{ER-FQ+GP}$

20. Calculate the fundamental magnitudes for the right helicoid given by $x = u \cos\phi, y = u \sin\phi, z = c\phi$.

21. Prove that $H^2\hat{n}_1 = (FM-GL)\vec{r}_1 + (FL-EM)\vec{r}_2$

22. Show that $H[\hat{n}\hat{n}_1\vec{r}_1] = EM - FL$.

23. State and prove Meunier's theorem.

24. Prove that if L,M,N vanish at all points of a surface, then the surface is a plane .

25. Calculate the fundamental magnitudes and the unit normal for the conoid $x = u \cos \phi, y = u \sin \phi, z = f(\phi)$ with u, ϕ as parameters .

26. Find the principal curvatures and the lines of curvature on the right helicoids $x = u \cos \phi, y = u \sin \phi, z = c\,\phi$.

27. Find the principal directions and principal curvatures on the surface $x = a(u + v), y = b(u - v), z = uv$.

28. Prove that the necessary and sufficient conditions that the parametric curves be asymptotic lines are $L = 0, M = 0, N = 0$.

29. Show that on the surface of revolution $x = u \cos \phi$, $y = u \sin \phi$, $z = f(u)$.

The asymptotic lines are $f_{11}\, du^2 + u f_1\, d\phi^2 = 0$.

30. Show that on the surface $z = f(x, y)$, the asymptotic lines are $r dx^2 + 2s\, dx\, dy + t\, dy^2 = 0$ and their torsions are $\pm \frac{\sqrt{s^2 - rt}}{1 + p^2 + q^2}$.

LINEAR ALGEBRA

Objective Types

(Tick the right answer)

1. Vectors specified by magnitude and direction and vectors belonging to a vector space are same. i) T ii) F

2. Set of functions 'f' satisfying the differential equation $\frac{d^2f}{dx^2}+\frac{df}{dx}$ = 0 is a vector space. i) T ii) F

3. Set of polynomials 'p', such that degree of p= 3 is a vector space.

i) T ii) F

4. R x R is a vector space. i) T ii) F

5. In a vector space V , (-1) (-u) = u for all u $\in$ V. i) T ii) F

6. Any subset of a vector space is a subspace. i) T ii) F

7. Subspace is a subset of vector space. i) T ii) F

8. If S_1 & S_2 are subspaces of a vector space V then $S_1 \cap S_2 = \phi$ the null space. i) T ii) F

9. S = { $\bar{0}$} ,$\bar{0}$ being the zero element of the vector space V is a subspace.

i) T ii) F

10. If $\alpha\bar{u}$ + $\beta\bar{v}$ $\in$S for every set of vectors$\bar{u}$and$\bar{v}$ and scalars $\alpha$and β, then S is a subspace. i) T ii) F

11. Every subspace of a vector space is another vector space. i) T ii) F

12. The derivatives of e^x form a vector space. i) T ii) F

13. The span of

    i. X-axis and Y-axis in V_3 is XY - plane. i) T ii) F

    ii. XY - plane and YZ - plane in V_3 is V_2. i) T ii) F

14. The span of the set $\{x, x^2, x^3, \ldots\}$ is 0. i) T ii) F

15. [Y-axis U Z-axis] = V_3 . i) T ii) F

16. For any subset A of a vector space V, [A] = [A+A] i) T ii) F

17. $A \subset [A]$ i) T ii) F

18. A set of two vectors in V_2 is always LI. i) T ii) F

19. A subset of LD set is never LD. i) T ii) F

20. A super set of a LI set is always LI. i) T ii) F'

21. If S and U are subspaces so are $S \cap U$ and S+U. i) T ii) F

22. Given S and W are subspace is S U W is a subspace. i) T ii) F

23. If $Z = S \cap W$, where S and W are subspaces , then $Z = \phi$, the null space. i) T ii) F

24. There exists a linear map such that T(0, 0) = (1,0,0,0)

i) T ii) F

25. T :P → P defined by T(P (x)) = x P (x) is not a linear map . i) T ii) F

26. A linear transformation cannot be defined from a real vector space to a complex vector space. i) T ii) F

27. Every constant map from one vector space to another is both one-one and onto. i) T ii) F

28. Every linear map from V_2 to itself is onto. i) T ii) F

29. Every linear map from V_3 to V_2 has an inverse. i) T ii) F

30. Inverse of non singular map is non singular . i) T ii) F

31. Every translation of $V_2 \to V_2$ is an isomorphism . i) T ii) F

32. The differential operator $D : P_n \to P_{n-1}$ has nullity zero. i) T ii) F

33. The matrix of identity map $I : U \to U$ relative to any pair of bases is the identity map. i) T ii) F

34. To every linear transformation there corresponds a unique matrix.

i) T ii) F

35. If T: $V_4 \rightarrow V_6$ is linear then the corresponding matrix is of dimension 6 x 4 . i) T ii) F

36. The matrix of the linear map T : $V_2 \rightarrow V_3$ is a square. i) T ii) F

37. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ is the matrix of identity transformation

I: $V_3 \rightarrow V_3$relative to bases B_1 = (e_1, e_2, e_3), B_x = {e_1, e_2) . i) T ii) F

38. If AB = I then A is invertible . i) T ii) F

39. For every integer n > 0 there is a non zero n x n matrix which is nilpotent. i) T ii) F

40. The range and kernel of a square matrix are of the same dimension.

i) T ii) F

LINEAR ALGEBRA

(Pass & Hons.)

LONG QUESTION:

1. Define a vector space and show that the n-triple of real numbers $(x_1, x_2, \ldots\ldots, x_n)$ constitute a vector space for addition and scalar multiplication defined as $(x_1, x_2, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots x_n + y_n)$; $\lambda(x_1, x_2, \ldots\ldots x_n) = (\lambda x_1, \lambda x_2, \ldots\ldots \lambda x_n)$

2. In any vector space V, prove the following results.

   a) $a\,\bar{o} = \bar{o}$, for every scalar a ,

   b) $0\,\bar{u} = \bar{o}$ for every $\bar{u} \in V$,

   c) $(-1)\bar{u} = -\bar{u}$ for all $u \in V$.

3. R^+ is a set of all positive real numbers. Addition and scalar multiplication are defined by $u + v = u.\,v$ for all $u, v \in R^+$ $au = u^a$ for all $u \in R^+$, a is a real scalar.

4. Define vector subspace. Prove that a subset *S* of a vector space V is a subspace iff

   i. if $u, v \in S \Longrightarrow u + v \in S$;

   ii. if $u \in S$, and scalar 'a', $au \in S$.

5. Let $W = \{ (x_1, x_2, \ldots, x_n) \in V_n^c : a_1 x_1 + \ldots\ldots + a_n x_n = 0 \}$, show that W is a vector space (a's are given numbers)

6. Which of the following sets are subspace of V_3 ?

   i. $\{(x_1, x_2, x_3): x_1 = 2x_2 \text{ or } x_3 = 3x_2\}$

   ii. $(p \in P : \text{degree of } p \le 3)$

   iii. $\{ f \in C(a, b) : f'(x_0) = 0 \text{ for all } x_0 \in (a, b)\}$

7. i) If S be a non empty subset of a vector space then [S], the span of S is a vector space . Prove this .

ii) [S] is the smallest subspace of V containing S . Prove this.

8. Given S = { (1, 2,1) , (1, 1, -1), (4,5,-2)}. Determine if the following vectors are in [S].

i) (1,1,0), ii) (1, -3 , 5)

9. Are the following polynomial in the set

$S = \{ x^3 , x^2 + 2x , x^2 + 2, 1- x \}$

i) $3x + 2$, ii) $x^3 + x^2 + 2x + 3$

10. If U and W are two subspaces of a vector space V, then U + W is a subspace of V ,and that $U + W = [U \; U \; W]$. Prove this.

11. Define direct sum of two vector spaces U and W are two subspaces of a vector space V and $Z = U \oplus W$. Then $Z = U \oplus W$ iff for $z \in Z$, $z = u \oplus v$, $u \in U$, $w \in W$. Prove this.

12. Define linear independence of a set of vectors. Prove that (1,0,1) , (1, 1, 0) ,(1, 1, -1) are L. I.

13. An ordered set of vectors $\{ v_1 , v_2 , ... , v_n \}$, $v_i \neq 0$ belonging to the vector space V is L.D. iff $v_k \subset [v_1 , v_2 , ... , v_{k-1}]$; $k \leq n$.

14. If a finite subset S of vector space V spans f, prove that linearly independent subset of S will also span f.

15. Test for linear independence of $s \in P$ the set of polynomials

i. $S = \{ x^2 - 1, \; x + 1, \; x - 1 \}$

ii. $S = \{ \ln x, \ln x^2 , \ln x^3 \}$

iii. $S = \{\sin x, \sin 2x, ..., \sin nx\}$, $s \in c\,[-\pi, +\pi]$,
n is a positive integer.

16. Prove that (a, b) and (c, d) are LD iff ad = bc.

17. Define basis for a vector space. In a vector space V if $\{ v_1 , v_2 ... v_n \}$ generates V and if $\{ w_1 , w_2 , ... , w_n)$ is LI, then show that $m \leq n$.

18. Basis for a given vector space consists of a fixed number of vectors. Prove this.

19. If S = (v_1 , v_2 ...v_p) is a subset of linearly independent vectors of a vector space V_n , $p < n$ then prove that S can be extended to become a basis for V_n.

20. If U and W are two subspaces of finite dimensional vectorspace V then show that $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$

21. Which of the following subsets of S form a basis for V ?

i. $S = \{(0, 0, 1)\ (1, 0, 1)\ (1, -1, 1)\}$; $v = v_3$

ii. $S = \{x - 1, x^2 + x - 1, x^2 - x + 1\}$; $v = p_2$

22. Extend { (3, -1, 2)} to two different bases for V_3.

23. Given $S_1 = \{(1, 2, 3), (0, 1, 2), (3, 2, 1)\}$ and

$S_2 = \{(1,-2,3), (-1, 1, -2), (1, -3, 4)\}$ determine the dimensions of

$[S_1] \cap [S_2]$ and $[S_1] + [S_2]$.

24. Find out a basis for a subspace U of V in the following cases ?

a. $U = \{p \in P_5 : p'' = 0\}, V = P_5$

b. $U = \{(x_1, x_2, x_3, \ldots, x_n) \in V_n : \sum_{i=1}^{n} a_i x_i = 0$, a_i's are scalars$\}$.

25. Find the coordinates of (1, 2, 1) and (2, 0, -1) w.r.t. the basis $S = \{(2,1,0), (2,1,1), (2,2,1)\}$.

26. Which of the following maps are linear ?

a. $T : V_1 \rightarrow V_2$ defined by $T(x) = (x, 2x, 3x)$

b. $T : V_3 \rightarrow V_3$ defined by $T(x, y, z) = (x^2 + xy, xy, yz)$

c. $T : P \rightarrow P$ defined by $T(p) = \phi(0)$

27. Determine whether there exists a linear map in the following cases. Give the general formula when it exists.

i. $T : V_3 \to V_2$ such that $T(1,2) = (3,0)$, $T(2,1) = (1,2)$

ii. $T : V_2 \to V_2$ such that $T(0,1) = (3,4)$, $T(3,1) = (2,2)$ and $T(3, 2) = (5,7)$

iii. $T : P_3 \to P_3$ such that $T(1 + x) = 1 + x$, $T(x) = 3$ and $T(x^2) = 4$

28. If $T : U \to V$ is a linear map . Then show that

a) R(T) is a subspace of V.

b) N(T) is a subspace of V.

c) T is one – one iff N(T) is zero subspace of U.

d) If $[u_1 \dots u_n] = U$ then $R(T) = [T(u_1), T(u_2), \dots T(u_n)]$

e) U is finite dimensional, then $R(T) \leq \dim (u)$.

29. Determine the range and kernel of the following linear maps.

i. $T : V_2 \to V_2$ defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$

ii. $T : V_4 \to V_4$ defined by $T(x_1, x_2, x_3, x_4) = (x_1 - x_4, x_2 + x_3, x_3 - x_4)$

30. Let $T : U \to V$ is a linear map. Then prove that

a) if T is one-one and $u_1 \dots u_n$ are LI vectors of U then $T(u_1), T(u_2) \dots T(u_n)$ are LI vectors of V.

b) if $v_1 \dots v_n$ are LI vectors of R(T) and $u_1, u_2 \dots u_n$ are such that $T(u_i) = v_i$, $i = 1 \dots n$, then $u_1, u_2 \dots u_n$ are LI.

31. If $T : U \to V$ be a linear map then show that $\dim R(T) + \dim N(T) = \dim U$

32. Let $T : U \to V$ be a linear map and $\dim U = \dim V = p$, then show that the following statements are equivalent.

a. T is non - singular .

b. T isone-one .

c. T transforms bases of U into bases for V.

d. T is onto .

e. $r(T) = P$,

f. $n(T) = 0$ and

g. T^{-1} exists

33. Define sum of two linear maps and multiplication of a linear map by a scalar. Prove that the set L(U,V) of all linear maps from U to V is a vector space.

34. Determine the matrices for the given linear transformations and the given bases B_1 and B_2 .

i. $T: V_2 \rightarrow V_2$, $T(x,y) = (x , -y)$, $B_1 = \{e_1, e_2\}$, $B_2 = \{(1, 1), (1,-1)\}$,

ii. $T : V_4 \rightarrow V_5$,

$T (x_1 ,x_2 ,x_3 ,x_4) = (2x_1 + x_2 ,x_2 - x_3 ,x_3 + x_4 , x_1, x_1 + x_2 + 3x_3 + x_4)$;

$B_1 = \{(1, 2,3,1) , (1,0,0,1), (1,1,0,0), (0,1,1,1)\}$ $B_2 = \{e_1 ,e_2 ,e_3 , e_4 , e_5\}$.

iii. $T : P_4 \rightarrow P_4$; $T (P(x))= \int_1^x p\,(t)dt.$

35. In the following cases obtain the transformation corresponding to the matrix and bases given :

i) $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$ $B_1 = \{(1, 1, 1, 2), (1-1, 0, 0), (0, 0, 1, 1),(0,1,0,0)\}$ $B_2 = \{(1, 2, 3), (1,-1, 1),(2, 1, 1)\}$

ii) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ $B_1 = \{(1, 1), (-1, 1),\}$ $B_2 = \{(1, 1, 1), (1,-1, 1), (0, 0, 1)\}$

36. If $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is the matrix of a linear map $T: V_2 \rightarrow V_2$ relative to standard bases, then find the matrix relative to standard bases.

37. i) If the inverse of a matrix exists , it is unique. Prove this.

ii) Determine A^{-1} when $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

38. If A = $\begin{vmatrix} \lambda_1 & 0 & \ldots.. & 0 \\ 0 & \lambda_2 \ldots\ldots. & & 0 \\ 0 & \ldots\ldots.. & & \lambda_n \end{vmatrix}$:B = $\begin{vmatrix} \mu_1 & 0 & \ldots.. & 0 \\ 0 & \mu_2 \ldots\ldots. & & 0 \\ 0 & \ldots\ldots.. & & \mu_n \end{vmatrix}$ Computer A^{-1}

39. A square matrix is non-singular iff its column vectors are L I.

40. Prove that $(A^T)^{-1} = (A^{-1})^T$.

41. If A is a square matrix then $A + A^*$ is Hermitian and $A - A^*$ is Skew-Hermitian.

42. Define rank of a matrix. Show that elementary row operationdo not alter the rank of a matrix.

43. Show that the row rank and column ranks of a matrix are equal.

44. Reduce the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ to row-reduced echelon form.

45. Prove that every elementary row(column) transformation. In a matrix is equivalent to pre-multiplication (post-multiplication) by the corresponding elementary matrix.

46. Find the rank of the following matrices :

i) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ ii) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

47. Show that every non-singular matrix can be expressed as the product of elementary matrices.

48. If x_1 and x_2 are two solution of AX = 0 , then $k_1 x_1 + k_2 x_2$ will be also a solution of AX = 0. Show this .

49. The number of independent solutions of a system AX = 0 of 'm' linear homogeneous equations in 'n' unknowns is (n - r), where r $=\rho(A)$. Prove this.

50. Discuss the nature of the solution of AX = 0, for different values of r. Given that A = $[a_{ij}]$ is of dimension m x n and $\rho(A) = r$.

51. The system of equations AX = B is consistent i.e. possesses at least one set of solutions iff $\rho(A) = \rho[A:B]$. Prove this. When is the solution unique?

52. Define eigen values and eigen vectors. Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$.

53. If x is an eigen vector of A corresponding to the eigen value λ^n, Prove that ;

a) x is an eigen vector of A^n corresponding to the eigen-value λ^n.

b) x is an eigen vector of g(A) corresponding to the eigen- value $g(\lambda)$.

54. If λ is an eigen value of A then show that λ is an eigen value of A^T and that A^{-1} an eigen value of A^{-1}.

55. Use cramer's rule to find out the solution of

$$x + y + 2z = 3$$
$$2x + 2y + 2z = 7$$
$$3x + 4y + 3z = 2$$

56. Using elementary transformations obtain the inverse of the matrix A given by $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

-The End-

MECHANICS

1. a) State and prove Triangle Law of Forces.

   b) State and prove the converse of Triangle Law of Forces.

2. a) State and prove the Polygon of Forces. Is its converse true ?

   b) State and prove Lamy's Theorem.

3. a) For the equilibrium of a system of particles, it is necessary that the vector sum of all forces, acting on all particles, is zero. Prove this. Also prove that in addition to this condition, it is also necessary that the sum of the moments of all external forces, about any line is zero.

   b) Under what conditions two different systems of forces will be equipollent ? Prove that a system of forces in the fundamental plane is plane equipollent to a single force applied at an arbitrary point in the plane, together with a couple.

4. a) Is the centre of mass of a system of particles the same point as that of the centre of gravity of the system? If the answer is yes, write the situations when it is true. Derive a formula for finding out the coordinates of the centre of mass of a system of particles, situated at given points.

   b) State and prove the theorems of Pappus for calculating mass-centres quickly.

5. a) Is there any difference between the Gravitational Force and the Force of Gravity ? Substantiate your answer with proper explanation.

   b) Write the Laws of Static and Kinetic Friction. A light ladder is supported on a rough ground, and leans against a smooth wall. How far up the ladder can a man climb without slipping of the ladder?

6. a) Write the mechanism of a Suspension Bridge with the Mathematical Derivations, satisfying the conditions of Flexible Cables.

   b) What is a common catenary ? Given the span and length, find out the maximum tension. Given the length and sag, find the span.

7. a) Find the normal and the tangential components of velocity and accelerations of a particle moving on a curve.

b) Derive the mathematical formulae or describing the velocity and the acceleration of a moving particle in plane polar co-ordinates.

8. a) What is a Hodograph? If a particle moves in a circle with constant speed, the hodograph is a circle, described with constant speed. Prove this .

b) Establish the principle of Linear Momentum, and the principle of Angular Momentum for a particle, moving in a plane.

9. State and prove the Principle of Energy and the Principle of Conservation of Energy.

10. a) The mass-centre of a system of particles moves like a particle, having a mass equal to the total mass of the system, acted on by a force equal to the vector sum of the external forces acting on the system. Prove this.

b) State and prove the D'Alemberts Principles.

11. a) Show that the path of a projectile is parabolic.

b) Find the time of flight, the ranges, the maximum height, altained by a projectile.

12. Find out the equation of the path of a projectile which moves, facing a resistance varying directly as its velocity.

13. Establish the formula i) $T = \frac{1}{2} I\omega^2$ ii) $I\omega = N$, and $\frac{1}{2} I. \omega^2 + V =$ constant for a particle, rotating about an axis with an angular velocity ω, having its moment of inertia I about the axis of rotation, where T is the kinetic energy, V is the potential energy and N is the total moment of the external forces about the axis of rotation for the particle.

14. a) Define Moment of Inertia, and give its physical interpretation. State and derive mathematical expressions for Principles of Parallel Axes

and Principle of Perpendicular Axes, applicable to find out Moment of Inertia.

b) Find out the Moment of Inertia of an elliptical plane lamina about a perpendicular axes, passing through the point of intersection of the major and the minor axes of the elliptic lamina.

15. a) For a rigid body, moving parallel to a fixed plane, the Kinetic Energy , T is given by $T = \frac{1}{2} . m V^2 + \frac{1}{2} I\omega^2$. Establish this where m is the mass of the body, V is the speed of mass centre, I is Moment of Inertia about the axis of rotation, passing through the mass centre, and W is the angular velocity.

b) What is a Compound Pendulum ? Find out its periodic time of oscillation for very small angular displacement (say, $\sin\theta = \theta$).

16. a) Can a Simple Pendulum be a special case of the Compound Pendulum ? Justify your answer with explanation.

b) Justify the principle of conservation of energy for a rigid body, moving parallaly to a fixed plane by establishing the formula $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + V = E$, where E is a constant, m is the mass of the body, v is the speed of the mass centre, I is the Moment of inertia about the mass centre, $\omega$is the angular velocity of the body, and V is the potential energy.

17. a) Derive the equations of motion of a cylinder rolling down an inclined plane .

b) What do you mean by a self propelled vehicle ? What external forces act on the vehicle when it is moving on the ground ? Clearly explain the mechanism of motion, by showing the actions of the forces with the help of a neat diagram.

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MODERN ALGEBRA

1. State True or False with reason.

   a. A group can be empty.

   b. $(2Z + 1, +)$ is a group.

   c. $(6Z, +)$ is a subgroup of $(9Z, +)$

   d. $(2^n : n \in Z)$ forms a group with respect to multiplication.

   e. $(Z_n, +)$ is a group.

   f. $(Z_6 - (0), .)$ is a group

   g. $\log : (R^+, .) \rightarrow (R, +)$ is an onto isomopehism.

   h. S_3, the group of permutation on 3 symbols, has no nontrival subgroup.

   i. S_3 has no normal sub group.

   j. $(2Z, +)$ is a normal subgroup of $(Z, +)$.

   k. Kernel of homomorphism of a group is a normal sub group.

   l. Every isomorphism is a homomorphism .

   m. If H is a subgroup of a group G then the coset form a group with respect to coset multiplication .

   n. $\frac{Z}{5Z}$ is isomorphic to Z_5 .

   o. $(Z, +, .)$ is a ring with Zero divisors.

   p. $(Z_6, +, .)$ is a ring with zero divisors.

   q. Every field is a ring without zero divisor.

   r. A field can contain a Zero divisor.

   s. Every ideal contains the unit element .

   t. No ideal contains the zero element .

   u. $(2Z, +, .)$ is an ideal of $(Z, +, .)$.

   v. Every integral domain is a field.

   w. The kernel of a ring homomorphism is always a two sides ideal .

x. (Z_7 , + ,.) is a field.

y. If R is a ring and a,b ∈ R, then $(a + b)^2 = a^2 + 2ab + b^2$.

2. Give an example of :-

a) A group with one element, another with exactly two elements yet another with exactly three elements, in each case specify the binary operation.
b) A group of order 11 with respect to multiplication.
c) A sequence of groups G_1 ,G_2 ,.... so that G_2 is a subgroup of G_1, G_3 of G_2, G_4 of G_3 and so on.
d) A cyclic group of order 7.
e) A normal subgroup of S_3.
f) Two subgroups whose union is not a subgroup.
g) Two subgroups whose union is a subgroup.
h) A ring with 5 zero divisors.
i) A field with 17 elements.
j) A ring which is not a field.
k) A ring which is not an integral domain.
l) A ring which is a division ring that is not a field.
m) A ring which has no nontrivial ideal.
n) A ring and an ideal of it such that the quotient ring is a field.
o) A ring that has no unit element .
p) A ring that is not commutative.
q) A ring homomorphism.
r) A ring isomorphism.
s) A group homomorphism.
t) A group isomorphism
u) Two groups that are isomorphic .
v) Two groups that are homomorphic but not isomorphic .
w) Two rings that are isomorphic .

3. Prove that :

i. Identity in a group is unique.
ii. Inverse of an element in a group is unique.
iii. Cancellation holds in a group .

iv. A group with two elements is abelian.

v. A group with three elements is abelian .

vi. A group with four elements is abelian.

vii. A group in which $(a\,b)^2 = a^2\,b^2$ is abelian.

viii. If G is a finite group, then there exists a positive integer n such that $a^n = e$, the identity .

ix. If in a finite set G a binary operation exists which is closed and associative and permits cancellation then G is a group.

x. If intersection of two left cosets is not empty then they are equal.

xi. If G is a group and $a \in G$, then $N(a) =\{x \in G: ax = xa \}$ is a subgroup of G .

xii. If G is a group then $Z = (z \in G : zx = xz$ for all $x \in G)$

xiii. If G is a finite group and $a \in G$, then order of a divides order of G.

xiv. If H is a subgroup of a finite group G and a, $b \in G$, a = b, then order of Ha = order of Hb

xv. If H is a subgroup of a finite group G and a, $b \in G$, a = b , then $Ha \cap Hb = \phi$.

xvi. If H is a subgroup of G, then H . H = H .

xvii. A group of prime order is Cyclic .

xviii. Intersection of two subgroups is a subgroup.

xix. If H is a subgroup and N a normal subgroup, then $N \cap H$ is normal subgroup .

xx. Every subgroup of an abelian group is normal .

xxi. If ϕ is a homomorphism of G into $\bar{G}$, then $\phi(e) = \bar{e}$, if e and $\bar{e}$,are identities in G and $\bar{G}$ respectively .

xxii. If ϕ is a homomorphism of G into $\bar{G}$, then $\phi(a^{-1}) = \{\phi(a)\}^{-1}$, for all $a \in G$.

xxiii. Range of every group homomorphism is a group .

xxiv. If$\phi : (R,+) \rightarrow (R\,,\,.)$ such that $\phi(x) = 2^x$, then ϕ is a homomorphism.

xxv. In every ring a0 = 0, when 0 is the additive identity and a is any element of the ring .

xxvi. If a, b $\in$ring R, then a(-b) = (-a)b = -ab

xxvii. $(Z_p , +,.)$ is a field, if P is prime.

xxviii. If U is an ideal of a ring R and $1 \in U$ then U = R .

xxix. If F is afield, its only ideals are (0) and F itself .

xxx. If R is a cummutative ring and $a \in R$ then $Ra = \{ ra : r \in R \}$ is an ideal of R.

xxxi. Prove that every subset of a finite group is a subgroup if it satisfies closer only.

xxxii. Prove that Congruent modulo a subgroup is an equivalence relation in the corresponding group .

xxxiii. State and prove Lagrange's Theorem.

xxxiv. If H and K are subgroups of a finite group G then $\frac{0\,(H).0\,(K)}{0\,(H \cap K\,)}$

xxxv. If H and K are subgroups of a group G then prove that HK is a subgroup if and only if HK = KH

xxxvi. Prove that any subgroup of a cyclic group is cyclic .

xxxvii. Prove that the subgroup N of G is a normal subgroup if and only if every left coset of N is a right coset of G.

xxxviii. If N is a normal subgroup of G then, prove that G/N is a group.

xxxix. Prove that Kernel of a homomorphism is a normal subgroup.

xl. If G is homomorphic to $\bar{G}$ with Kernel K then prove that G/K is isomorphic to $\bar{G}$.

xli. Prove that every finite integral domain is a field.

xlii. If I is an ideal of a ring R, then prove that R/I is a ring.

xliii. If a ring R is commutative with unit element and with (0) and R as the only ideals, then prove that R is a field.

xliv. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R iff R/M is a field.

Numerical Analysis

Long Questions

1. Convert $(0.7)_{10}$ to the corresponding binary fraction. If only 7 bits are retained in the binary fraction then find the corresponding decimal number and also find the round off error.
2. What is a floating point number ? Subtract the floating point 0.36132346×10^7 from 0.36143447×10^7 and express the result in normalized floating point form.
3. Obtain a second degree polynomial approximation to $f(x) = (1 + x)^{\frac{1}{2}}, x \in [0,0.1]$ using Taylor series expansion about x = 0 , use the expansion to approximate f(0.5) and bound the truncation error.
4. Find the smaller root of the equation $x^2 - 400x + 1 = 0$ using 4 digit arithmetic.
5. Compute the midpoint of the numbers a = 4.568 and b = 6.762, using four digit arithmetic.
6. Determine the real root of the eqn, $x^3 - x - 1 = 0$, correct to six decimals using Bisection method.
7. Perform five iterations of Regula-Falsi method to determine the root of the equation $\text{Cos}x - xe^x = 0$.
8. Discuss the convergence of Newton - Raphson method.
9. Perform five interations of Newton - Raphson method to find the real root of the equation 2x – sinx - 5 = 0.
10. If $\phi(x)$ is a continuous function in the interval [a, b] that contains the root and $|\phi'(x)| \leq C < 1$ in this interval, then for any choice of $x_0 \in [a, b]$, the sequence (x_k) determined from $x_{k+1} = \phi(x_k), k = 0, 1, 2, \dots,$ convergence to the root ζ of $x = \phi(x)$. Prove this.
11. Derive the Bisection method to find the real roots of the given equation. Derive the process by which approximate number of iterations required will be determined if the permissible error is $\in$.
12. Derive the secant method to find the real root of the given equation and give the geometrical interpretation of the method.

13. Define rate of convergence of an iterative method and prove that the rate of convergence for the secant method is 1.618.
14. Calculate the positive root of the equation $x^2 + 2x - 2 = 0$ correct up to 2- significant figures by Newton - Rapson method.
15. Establish Gregory - Newton Forward difference interpolation formula.
16. Establish Gregory -Newton Backward difference interpolation formula.
17. Find the unique polynomial of degree 2 or less such that $f(0) = 1, f(1) = 3, f(3) = 55$. Using i) Lagrange's interpolation, ii) The iteratated interpolation.
18. Use the Lagrange and the Newton-divided difference formulas to calculate f(3) from the following table

X	0	1	2	4	5	6
f(x)	1	4	15	5	6	19

19. For the following data, calculate the differences and obtain the forward interpolation polynomial. Interpolate at x = 0.35

X	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

20. Polynomial p(x) coincides with the function f(x) at x_0 and x_4 and it deviate at all other points, in the interval (x_0, x_1) . Then derive an expression for truncation error E_1 (f, x) for.
21. Prove that $\Delta^n f(x_i) = \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!} f_{i+n-k}$, where Δ is the forward difference operator.
22. Find the unique polynomial p (x) of degree 2 or less such that p (1)= 1, p (3) = 27, p (4) = 64 using Newton- Divided difference formula.
23. Establish Lagrange's interpolation formula.
24. Derive Trapezoidal rule to find the approximate value of a given integral and also derive formula to obtain the corresponding error.
25. Derive Simpson's rule to find the approximate value of a given integral and also derive a formula to obtain the corresponding error.
26. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using Trapizoidal rule. Obtain a bound for the corresponding error.
27. Apply Simpson's rule to evaluate $\int_0^{\frac{\pi}{2}} sinx\, dx$ and obtain a bound for the corresponding error.

28. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss- Legendre three points rule correct to six decimal places.
29. State and explain briefly the Gaussian numerical integration technique, illustrating in particular the case, n = 2 and derive the corresponding formula.
30. Derive Euler Method to obtain numerical solution of $\frac{du}{dt} = f\,(t,u), u\,(t_0) = \tau$ and derive a formula for the corresponding error.
31. Use Euler method to solve numerically the initial value problem $u' = -2tu^2$, U(0) = 1 with u = 0.1 in the interval [0, 1].
32. Use Euler method to solve numerically the initial value problem u'= t +u, u(0) = 1 with h = 0.2 in the interval [0, 1].

Short Questions:

1. Round off the number 2.36425 to 4 significant figures and find the relative percentage of error.
2. Convert $(1101001.1110011)_2$ to corresponding octal number.
3. Convert $(1110001101.1110011001)_2$ to the corresponding hexadecimal number.
4. Convert $(703.65)_8$ to its corresponding decimal number.
5. Convert $(703.65)_8$ to its corresponding binary number.
6. Define a floating point number and state when it will be normalised.
7. Find the sum of $123 * 10^3$ and $.456 * 10^2$ and write the result in three digit mantisa using chopping method.
8. If we take $x = 3.14$ instead of 3.14156 find relative percentage error.
9. Explain geometrically the secant method.
10. Find the order of convergence of Newton-Raphson method.
11. If $\phi(x)$ is iteration function, C_k is the error in the kth Iterate this show that $\varepsilon_{k+1} = a_1\varepsilon_k + a_2\varepsilon_k^2 + 0\,\langle\varepsilon_k^3\rangle$
    And for first order method $\underset{k}{\varepsilon} = a_1^k\,\varepsilon_0\ \langle k = 0, 1, \ldots\ldots\rangle$
12. Writ the statement of the theorem on which the bisection method for finding roots of an equation is based.
13. State the error for Newton - Raphson process.

14.When a secant method will be called as Regula - Falsi method?

15.Obtain a polynomial approximation $P(x)$ to $f(x) = e^{-x}$ using the Taylor's expansion about x_o and determine x when the error in $P(x)$ obtain from the first four terms only is to be less then 10^{-6} after sending for $0 \leq x \leq 1$.

16.Derive linear Lagrange interpolating polynomial and prove that

$$l_i(x_j) = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases}$$

17.Derive iterated linear interpolating polynomial.

18.Use sin (0.1) = 0.09983 and sin (0.2) = 0.19867, find an approximate value of sin(0.15) by Lagrange interpolation. Obtain a bound on the truncation error.

19.Determine the stepsize 'h' to be used in the tabulation of f(x) = cosx in the interval [1,4], so that the linear interpolation will be correct to four decimal places.

20.Prove that :-

21.i) $\Delta f_i = \nabla f_{i+1} = \sigma f_{i+1/2}$ ii) $\Delta = \nabla E$.

22.Prove that : $\mu = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$ ii) $\Delta = E - 1$

23.Prove that :

i) $\nabla = 1 - E^{-1}$ ii) $\sigma = E^{1/2} - E^{-1/2}$

24.Prove that = $\sigma = E^{1/2} - E^{-1/2} = \Delta(1 + \Delta)^{-1/2} = \nabla(1 - \nabla)^{-1/2}$

25.Write the Gregory Newton-forward interpolating polynomial of degree n and also write the corresponding error term.

26.Write the Gregory Newton-Backward interpolating polynomial of degree n and also write the corresponding error term.

27.Prove that $f[x_0, x_1, \ldots x_n] = \frac{1}{n!h^n}\Delta^n f_0$

28.Prove that $f[x_0, x_1, \ldots x_n] = \frac{1}{n!h^n}\nabla^n f_n$

29.Derive an expression for $f[x_0, x_1, \ldots x_n]$ in terms of central difference operatorσ.

30.If $f(x) = e^{ax}$, then show that $\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$.

31.Prove that $\Delta(\frac{f_i}{g_i}) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i\ g_{i+1}} =$

32.Show that : $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

33. What is interpolating polynomial ?

34.Write the relationship that exists between forward and backward differences.

35.Give the geometrical interpretation of Trapezoidal formula and write its error term.

36.What is the error in approximating the integral $\int_1^2 x\, dx$ by Simpson's rule.

37.Derive the error term R_2 in the Simpson's rule.

38.Derive the error term R_1 in the Trapezoidal rule.

39.When can a numerical method of solving differential equation is convergent ?

40.State Simpson's rule for numerical integral .

41.Show that $\frac{(u_{j+1} - u_j)}{h} = f\,(t_j, u_j)$, where $\frac{du}{dt} = f(t, u)$ and u_j is a number which is an approximation to the value of u(t) at the point t_j.

42.Derive an expression for absolute value of the truncation error in the Euler's method .

43.Evaluate u(0.4) using Euler's method from $u' = -2tu^2$, u(0) = 1.

44.What is the first order Adams-shforth method and where it is applied?

Operations Research

(Pass and Hons)

1. Show that a necessary and sufficient condition for the existence and non-degeneracy of all the basic solutions of AX = b is that every set of m columns of the augmented matrix Ab = [A , b] is linearly independent.

2. Define a hyper plane and prove that a hyper plane is a convex set.

3. Show that $S = \{ f(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4 \} \subset R^2$ is a convex set.

4. Solve graphically the following LPP : maximize $Z = 2x_1 + 3x_2$ subject to

$$x_1 + x_2 \leq 1, \quad 3x_1 + x_2 \leq 4, \quad (x_1, \quad x_2 \geq 0)$$

5. Prove that every extreme point of the convex set of feasible solutions is a BFS.

6. Prove that the objective function of a LPP assumes its minimum value at an extreme point of the convex set X generated by the set of all feasible solutions.

7. Solve graphically the following LPP. Minimize $Z = 2x_1 + x_2$ subject to

$$5x_1 + 10x_2 \leq 50, \quad x_1 + x_2 \geq 1, \quad x_2 \leq 4, \; x_1, x_2 \geq 0)$$

8. Prove that the set of all feasible solutions of a LPP is a convex set .

9. Compute all the non-degenerate basic feasible solutions of the following equation

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{x_3}{2} + x_5 = 2$$

is the solution [1 , 1/2 ,0,0,0]a basic solution ?

10. Prove that every basic feasible solution of the system AX = b , $X \geq 0$ is an extreme point of the convex set of feasible solutions.

11. Find all basic solutions of the system of linear equation

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

12. Given a LPP explain under what conditions.

i) A BFS can be improved

ii) The solution is unbounded.

13. Define :- i) Basic solution. ii) basic feasible solution.

iii) convex set. iv) convex cone.

v) convex hull of a set. vi) convex polyhedron.

vii) non-degenerate solution. viii) optimum solution.

14. Solve graphically the following LPP:

i) Max Z = $6x_1 + 11x_2$ such that $2x_1 + x_2 \leq 104$, $x_1 + 2x_2 \leq 76$ and $x_1 , x_2 \geq 0$.
ii) Min Z = $3x_1 + 5x_2$ such that $- 3x_1 + 4x_2 \leq 12$, $2x_1 - x_2 \geq -2$, $2x_1 + 3x_2 \geq 12$, $x_1 \leq 4$, $x_2 \geq 2$, $x_1 , x_2 \geq 0$.

15 Define :-i) Hyper plane. ii) Extreme point of a convex set. iii) convex function.

16. Prove that a hyper plane is a convex set.

17. Prove that the intersection of two convex sets is a convex set. Is this result true for a finite number of sets?

18. Prove that the extreme points of the convex sets of feasible solutions are finite in number.

19. If the convex set of the feasible solutions of AX = b , $X \geq 0$ is a convex polyhedron, then at least one of the extreme points gives an optimal solution .

20. A hyper plane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$

Find in which half space do the following points (-6 , 1 , 7 , 2) and (1,2,-4,1) lies .

21. Prove that the following sets are convex.

i) The interior and the edge of a triangle.

ii) The interior of circle.

iii) A rectangle surmounted by a semi-circle.

Which of the above are convex polyhedron ?

22. Which of the following sets are convex, if so why ?

i)$X = \{[x_1, x_2]: x_1, x_2 \leq 1,\ x_1 \geq 0,\ x_2 \geq 0\}$

ii) $X = \{[x_1, x_2]: x_2 - 3 \geq -x_1^2,\ x_1 \geq 0,\ x_2 \geq 0\}$

iii) $X = \{[x_1, x_2]:\ x_1 \geq 2,\ x_2 \leq 3\}$

23. Determine the convex hull of the following sets A :

i) $A = \{(x_1, x_2):\ x_1^2 + x_2^2 = 1\}$

ii) $A = \{(x_1, x_2)\}$

24. State and prove the fundamental theorem of linear programming.

25. If a linear programming problem max Z = cx s.t. AX = b, X $\geq$ 0 has at least one feasible solution, then prove that it has at least one basic feasible solution also.

26. If the LPP max Z = cx s.t. AX = b, x $\geq$ 0 has feasible solution, then show that at least one of the B.F. solution will be optional.

27. Solve the L.P. problem : maximize Z = $3x_1 + 5x_2 + 4x_3$ such that $2x_1 + 3x_2 \leq 8$, $2x_2 + 5x_3 \leq 10$,

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

And

$$x_1, x_2, x_3 \geq 0$$

28. if $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible solution of the LPP

max $Z = x_1 + 2x_2 + 4x_3$

such that $2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

$x_1, x_2, x_3 \geq 0$

then find a BFS.

29. Solve the following LPP by simplex method.

max $Z = 5x_1 + 3x_2$

such that $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$

30. Define with examples slack and surplus variables and state the LPP after introducing these.

31. Solve the following LPP by simplex method :

i) Min $Z = x_1 + x_2$
such that $2x_1 + x_2 \geq 4$
$x_1 + 7x_2 \geq 7$
$x_1, x_2 \geq 0$.

ii) max $Z = 3x_1 + 2x_2 + x_3$
such that $-3x_1 + 2x_2 + 2x_2 = 8$
$-3x_1 + 4x_2 + x_3 = 7$
and $x_1, x_2, x_3 \geq 0$.

iii) max $Z = 2x_1 + x_2$
such that $x_1 - x_2 \leq 10$
$2x_1 - x_2 \leq 40$
and $x_1, x_2 \geq 0$

iv) max $Z = 2x_1 + 3x_2$
such that $-x_1 + 2x_2 \leq 4$, $x_1 + x_2 \leq 6$.

$x_1 + 3x_2 \leq 9,$ $\quad$ x_1, x_2 unrestricted.

v) min $Z = 2x_1 + 9x_2 + x_3$

such that $x_1 + 4x_2 + 2x_3 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

$X_1, x_2, x_3 \geq 0$.

vi) max $Z = 2x_1 - x_2 + x_3$

such that $x_1 + x_2 - 3x_3 \leq 8$

$4x_1 - x_2 + x_3 \geq 2$

$2x_1 + 3x_2 - x_3 \geq 4$

And $x_1, x_2, x_3 \geq 0$.

vii) max $Z = 3x_1 + 2x_2$

such that $2x_1 + 2x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$.

32. Explain the LPP giving two examples. When a mathematical programming problem is called a "linear" one ? What is an unbounded solution to a LPP ?

33. What do you understand by "graphical method" ? Give its limitations.

34. Mathematics of Operation research is mathematics of optimization - discuss.

Partial Differential Equations

(+3 Hons.) (PDE)

1. Form P.D.E's by eliminating the arbitrary constants.
   i. $x^2 + y^2 + (z-c)^2 = a^2$;
   ii. $z = ax^2 + bxy + cy^2$.

2. Eliminate the arbitrary functions from the following through partial derivatives.
   i. $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$;
   ii. $y = yf(x) + x\,g(y)$
   iii. $z = f\left(\frac{xy}{z}\right)$

3. Define a linear partial differential equation. $Is\, z^2 = (xp - yq)$ a linear equation ? Solve :
   i. $x^2 p + y^2 q = (x+y)z$
   ii. $\cos(x+y)p + \sin(x+y)q = z$

4. Obtain the solution of the following PDE'S :
   i) $p + 2q = 5z + \cos(y - 2x)$,
   ii) $x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) + z\left(\frac{\partial u}{\partial z}\right) = au + \left(\frac{xy}{z}\right)$.

5. Use Charpit's method to solve the following PDES
   i. $2(z + xp + yq) = yp^2$
   ii. $(p+q)(px+qy) = 1$

6. Obtain complete integral of
   i. $z = px + qy + p^2 + q^2$,
   ii. $p^2 + px + q = z$

7. Obtain complete integral of
   i. $pq = 1$,
   ii. $(1 - x^2)yp^2 + x^2 q = 0$

8. Obtain complete integral of :
   i. $q(p^2 z + q^2) = 4$,
   ii. $q^2 y^2 = z(z - px)$

9. Obtain complete integral of :
   v) $p^2 + q^2 = x + y$
   vi) $x^2 p^2 = yq^2$

10. Solve the following equations :

i) $z = px + qy + 3(pq)^{\frac{1}{3}}$

ii) $4xyz = pq + 2px^2y + 2qxy^2$

11. Obtain the general solution of the differential equation $xzp + yzq = xy$. Also find the integral surface which passes through the curve $y^2 = 4ax$, $z = 0$

12. Find the complete integrals of :

i) $p + q = Sinx + Siny$,

ii) $z^2 (p^2 x^2 + q^2) = 1$

13. Solve:

i) $(D^2 - D'^2)z = 0$,

ii) $(D^4 + D'^4 - 2D^2 D'^2) z = 0$

Where $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$

14. Solve $(D - 2D')(D - 2D' + 1)(D - D'^2) z = 0$

15. Solve the following :

i) $(D^2 - D') z = A \cos(lx + my), l, m$ are constants.

ii) $(D^2 - D'^2 + D + 3D' - 2) z = e^{x-y} - x^2 y$.

16. Solve the following :

i) $\left(x^2D^2 - 2xyDD' + y^2 D'^2 - xD + 3yD'\right) z = 8\frac{y}{x}$

ii) $x^2r - y^2t = xy$, where $r = \frac{\partial^2 z}{\partial x^2}, t = \frac{\partial^2 z}{\partial y^2}$

17. Reduce to canonical form and then solve :

i) $r + 2s + t = 0$

ii) $x(xy - 1)r - (x^2y^2 - 1)s + y(xy - 1)t + (x - 1)P + (Y - 1)q = 0$

18. Solve :

i) $t = x \sin(xy)$ ii) $s = 4xy + 1$

19. Using method of separation of variables solve :

i) $\left(\frac{\partial^2 u}{\partial x^2}\right) - 2\left(\frac{\partial u}{bx}\right) + \left(\frac{\partial u}{\partial y}\right) = 0$,

ii) $\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial u}{\partial y}\right) = 2u$, Given that $u(O, y) = 0$ and

$\frac{\partial u}{\partial x}(0, Y) = 1 + e^{-3y}$

20. Solve the following using Monge's method

i) $x^2r + 2xys + y^2 t = 0$

ii) $(e^x - 1)(qr - ps) = pqe^x$

p, q, r, s have their usual meaning .

21. Solve the Following :-

i) $(D^2 + DD' + D' - 1)\ z = 4\ sinhx$

ii) $(D^2 - 4DD' + 4D'^2)\ z = xe^{2x+1}$

22. Solve :

i) $xy^2s = 1 - 2x^2\ y$,

ii) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y}$

(use separation of variable method)

23. Form the P.D.E's by eliminating the arbitrary functions

i) $V = \frac{1}{r}[\ \mathrm{l}(r - at\ )\mathrm{l} + F(\ r + at)\ ]$

ii) $f(x^2 + y^2\ ,\ z - xy) = 0.$

24. Solve :

i) $\frac{\partial^2 u}{\partial x \partial y} = 0$, subject to $u\ (O, y) = y,\ u\ (x, 0) = sinx$

ii) $\frac{\partial^2 z}{\partial x^2} = a^2\ z$, given that $\frac{\partial z}{\partial x} = a\ siny$ and $\frac{\partial z}{\partial y} = 0$ when $x = O.$

25. Obtain the differential equations of all spheres whose centres lie on the z-axis .

26. Find the general integral of the following

i) $p\ tanx + q\ tany = tanz$

ii) $p - q = log(x + y)$

27. Find the complete integral of

i) $p^3 - q^3 = 0,$ ii) $z = p^2x + q^2y$

Probability Theory

(+3 Degree Hons. Course)

1. Two players A and B play a series of games in which the probability of each winning a single game is ½ , irrespective of the outcomes of other games. Each player gains a "point" when he wins a game, and nothing when he loses. Suppose that they stop playing when A needs 2 more points, and B needs 3 more points to win the stake. How should they divide it fairly ?

2. If six dice are rolled, what is the probability that all show different faces ?

3. Prove that $P(AUB) = P(A) + P(B - AB) = P(A) + P(B) - P(AB)$

4. What is the probability of the set of numbers {1, 2, 3 .... 120) divisible by 3, not divisible by 5, and divisible by either 4 or 6 ?

5. Group A, B, C have respectively 57, 49, 43 members . A and B have 13, A and C have 7, B and C have 4 members in common; and there is a lone guy which belongs to all three groups. Find the total number of people in all three groups.

6. Six mountain climbers decide to divide into three groups for the final assault on the peak. The groups will be of size 1, 2, 3 respectively and all manners of deployment are considered. What is the total number of possible grouping and deploying ?

7. Prove that the sum of the n^{th} row in Pascal's triangle is 2^n.

8. If a deck of poker cards are thoroughly shuffled, then what is the probability that the four aces are found in a row ?

9. Six dice are rolled. What is the probability of getting three pairs ?

10. Fifteen new students are to be evenly distributed among three classes. Suppose that there are three whiz-kids among the fifteen. What is the probability that each class gets one ? One class gets them all ?

11. If $\emptyset$ is a function of two variables and X and Y are random variables $W \to \emptyset\,(X(w), Y(w)\,)$ is also a random variable, which is denoted more concisely as $\emptyset(X, Y)$. Prove this.

12. The cost of manufacturing a certain Book is Rs.3/- per book up to 1000 copies, Rs.2/- per copy between 1000 and 5000 copies, and Rs.1/- per copy afterwards. Suppose 1000 copies were printed initially and the price was Rs.5/- per book. X books will be sold. Compute the profit or loss

from the sales called as Y. What is the probability that the book is a financial loss and the profit will be at least Rs.10.000/- ?

13. Explain Distribution function of X and prove that

$P(a < X \leq b) = F_x(b) - F_x(a).$

14. A perfect coin is tossed repeatedly until a head turns up. Let X denotes the number of tosses it takes till a head turns up. If $X = n$ What is the expectation of X ?

15. A chord is drawn at random in a circle. What is the probability that its length exceeds that of a side of an inscribed equilateral triangle ?

16. A perfect die is thrown twice. Given that the total obtained is 7, what is the probability that the first point obtained is $k, 1 \leq k \leq 6$?

17. Suppose that $\Omega = \sum_n A_n$ is a partition of the sample space into disjoint sets. Then for any set B we have $P(B) = \sum_n P(A_n) P(\frac{B}{A_n})$. Prove this.

18. Suppose that the sun has risen 'n' times in succession. What is the probability that it will rise once more ?

19. $\Omega = \sum_n A_n$ is a partition of the sample space. For any set B prove that $P\left(\frac{A_n}{B}\right) = \frac{P(A_n) P\left(\frac{B}{A_n}\right)}{(\sum_n P(A_n) P\left(\frac{B}{A_n}\right)}$

20. The family dog is missing after picnic. The followings are suggested.

i) It has gone home.

ii) It is still worrying that big bone in the area.

iii) It has wandered off into the woods.

What is the probability that dog will be found in the park.

21. Prove that $E(X + Y) = E(X) + E(Y)$ where X, Y and $X + Y$ are summable.

22. There are N coupons marked 1 to N in a bag. Draw one coupon after another with replacement. What is the expected number of drawings to get 'r' different coupons.

23. For arbitrary events $A_1, A_2, \ldots A_n$ we have

$$P\langle U_{j=1}^{n} A_j \rangle = \sum_j P(A_j) - \sum_{j,k} P(A_j A_k) + \sum_{j,k,l} P(A_j A_k A_l) + \ldots\ldots + (-1)^{n-1} P(A_1 A_2 \ldots A_n)$$

when the indices in each sum are distinct and range from 1 to n. Prove this.

24. If X and Y are independent summable random variables, then $E(XY) = E(X)\,E(Y)$. Prove this.

25. If X and Y are independent , and both have finite variances, then $\sigma^2(X + Y) = \sigma^2(X) + \sigma^2(Y)$. Prove this.

26. For any two constants a and b, $-\infty < a < b < +\infty$

$\lim_{n\to\infty} P\left(a < \left(\frac{S_n - np}{\sqrt{npq}}\right) \leq b\right) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-\frac{x^2}{2}}\,dx$ Prove this.

27. The total number of arrivals in a time interval of length 't' has the Poisson distribution π (at) for each $t > 0$ .Prove this

28. Consider the number of arrivals in two disjoint time intervals, $X_1 = N(s_1, s_1 + t_1)$ and $X_2 = N(s_2, s_2 + t_2)$.

What is the probability that the total number $X_1 + X_2 = n$?

29. Let X_j be independent random variables with Poisson Distribution $\pi(\alpha_j), 1 \leq j \leq n$. Then prove that $(X_1 + X_2 + \ldots.. + X_n$ has Poisson distribution $\pi\,(a_1 + a_2 + \ldots. + a_n)$.

30. A perfect die is rolled 100 times . Find the probability that the sum of all parts obtained is between 330 and 380.

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